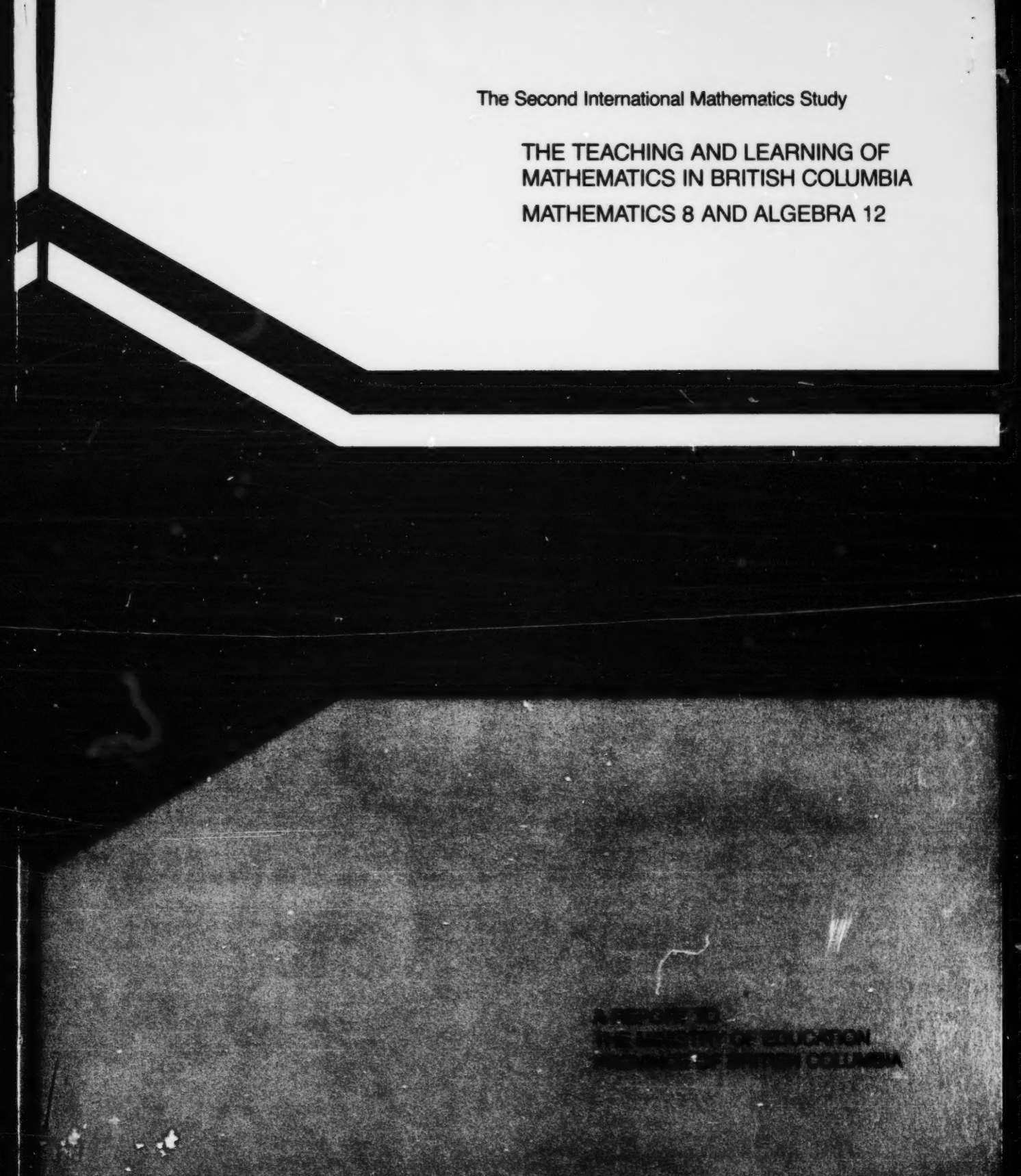
The Second International Mathematics Study

THE TEACHING AND LEARNING OF MATHEMATICS IN BRITISH COLUMBIA

MATHEMATICS 8 AND ALGEBRA 12



The Second International Mathematics Study

THE TEACHING AND LEARNING OF MATHEMATICS IN BRITISH COLUMBIA

MATHEMATICS 8 AND ALGEBRA 12

David F. Robitaille
Thomas J. O'Shea
Michael K. Dirks

Learning Assessment Branch
Ministry of Education
Province of British Columbia

April 1982

Canadian Cataloguing in Publication Data

Robitaille, David F.

The teaching and learning of mathematics in British Columbia

At head of title: The second international mathematics study.
ISBN 0-7719-8991-1

1. Mathematics - Study and teaching (Secondary) - British Columbia. 2. Mathematical ability - Testing. 3. Curriculum evaluation. I. O'Shea, Thomas. II. Dirks, Michael K. III. British Columbia. Learning Assessment Branch. IV. Title. V. Title: The second international mathematics study.

QA14.C32B754 510'.7'12711 C82-092200-5

The Second International Mathematics Study

THE TEACHING AND LEARNING OF MATHEMATICS IN BRITISH COLUMBIA: MATHEMATICS 8 AND ALGEBRA 12

Advisory Committee

Alan Taylor (Chairman)	Learning Assessment Branch Ministry of Education
David Bateson	Learning Assessment Branch Ministry of Education
Mary Cooper	Educational Planning & Research B.C. Research
Hugh Elwood	Teacher Burnaby School District
Barbara Holmes	Educational Planning & Research B.C. Research
Marilyn Kienas	Teacher Maple Ridge School District
Diane McKendrick	Trustee Powell River School District
Tomo Naka	Principal Nelson School District
Brin Nevile	Teacher Prince George School District
Art Olson	Director of Instruction Qualicum School District
David Robitaille	Faculty of Education University of British Columbia
Jim Vance	Faculty of Education University of Victoria
Robert Wilson	Learning Assessment Branch Ministry of Education
Sophie Zonnis	Teacher Central Okanagan School District

The Second International Study of Mathematics

THE TEACHING AND LEARNING OF MATHEMATICS IN BRITISH COLUMBIA:
MATHEMATICS 8 AND ALGEBRA 12

Technical Sub-Committee

Robert Wilson (Chairman)	Learning Assessment Branch Ministry of Education
David Bateson	Learning Assessment Branch Ministry of Education
Mary Cooper	Educational Planning & Research B.C. Research
Barbara Holmes	Educational Planning & Research B.C. Research
Thomas O'Shea	Faculty of Education Simon Fraser University
David Robitaille	Faculty of Education University of British Columbia
James Sherrill	Faculty of Education University of British Columbia
Alan Taylor	Learning Assessment Branch Ministry of Education

TABLE OF CONTENTS

		Page
PREFACE		vii
1. BACKGROUND AND STRUCTURE OF THE STUDY		1
1.1	Purpose of the study	2
1.2	Structure of the study	3
1.3	Framework for the study	5
1.4	Rationale for B. C. participation	8
1.5	The B. C. sample	8
1.6	Return rates	9
2. THE MATHEMATICS CURRICULUM		15
2.1	Development of the international item grids	15
2.2	Curricular comparisons	19
2.3	Item development and validation	23
2.4	Appropriateness of the item pools	25
3. THE TEACHING OF MATHEMATICS: GRADE 8		28
3.1	Treatment of content topics	29
3.2	Concept interpretations and teaching methods	36
3.3	Use of instructional time	44
3.4	Use of instructional resources	46
3.5	Influences in content decisions	48
3.6	Variety of interpretations and strategies	54
3.7	Influence of textbooks	56
4. THE TEACHING OF MATHEMATICS: GRADE 12		58
4.1	The teaching of analytic geometry	61
4.2	The teaching of trigonometry	65
4.3	The teaching of algebra	70
4.4	Formulas, their justification and use	76
4.5	Use of instructional time and other resources	78
4.6	Influences on content decisions	80
4.7	Organization of instruction	84
5. STUDENTS' ACHIEVEMENT: GRADE 8		87
5.1	Suitability of items to the B. C. curriculum	88
5.2	Core test results	93

5.3 Gains in achievement 106
5.4 Opportunity-to-learn and achievement 109
5.5 Classroom-based gain scores 111
5.6 The rotated forms 113
5.7 Summary 114

6. STUDENTS' ACHIEVEMENT: GRADE 12 115

6.1 Non-curriculum items 118
6.2 Curriculum items 120
6.3 Achievement by cognitive level 140

7. STUDENTS' ATTITUDES: GRADES 8 AND 12 141

7.1 Description of the instruments 141
7.2 Mathematics in school 141
7.3 Mathematics as a process 149
7.4 Parental support for mathematics 152
7.5 Mathematics and myself 153
7.6 Mathematics in society 155
7.7 Mathematics and gender 156
7.8 Calculators and computers 157

8. MAJOR FINDINGS AND IMPLICATIONS 160

APPENDICES 167

A. Authors of the report 167
B. Participating teachers - Grade 8 168
C. Participating teachers - Grade 12 171
D. Sample questionnaire - Grade 8 174
E. Sample questionnaire - Grade 12 197
F. Items on rotated forms - Grade 8 214
G. Prescribed curriculum - Mathematics 8 229
H. Prescribed curriculum - Algebra 12 234

PREFACE

This report contains an analysis and discussion of some of the major findings of the Second International Mathematics Study as it was conducted in British Columbia. Despite its size, the report deals with only a portion of the results of the study, and much work remains to be done. For example, no attempt has been made as yet to use the data to relate aspects of the methodology used by teachers of mathematics to student outcomes. It is hoped that the resources needed to carry out those analyses will become available in the near future.

Participation in this project called for a significant contribution of time and effort on the part of approximately two hundred teachers of mathematics and their students. The names of those teachers are listed in the appendices, and they deserve to be congratulated for the high level of professionalism and dedication they exhibited throughout the study.

Thanks are also due to a number of individuals who worked extremely hard to ensure the success of this project which seemed to grow in complexity and difficulty with each passing day. This group includes Mary Cooper and Barbara Holmes of B. C. Research, Harold McAllister, a graduate student at Simon Fraser University, Alan Taylor of the Ministry of Education, and Bob Wilson of Queen's University who was with the Ministry of Education at the time. Rita Espinosa, Patricia Fairey, Eileen Mallory, Lynda Robitaille, and John Wiebe provided valuable assistance in the documentation of the data and the preparation of this report.

Officials of the Ministry of Education have been very supportive of this project from the time that B. C. participation was first proposed almost four years ago. I would like to thank, in a special way, Jim Carter, the Deputy Minister, Gerry Mussio, Nancy Greer, and Joyce Matheson for their support and encouragement.

David F. Robitaille

University of B. C.

CHAPTER 1

BACKGROUND AND STRUCTURE OF THE STUDY

During the 1980-81 school year, several thousand Grade 8 and Grade 12 mathematics students from British Columbia, along with their mathematics teachers, participated in the Second International Mathematics Study. More than twenty countries took part in various phases of the study, with Canada being represented through the participation of British Columbia and Ontario. A list of the participants is given below:

Australia	Israel
Belgium (Flemish)	Ivory Coast
Belgium (French)	Japan
Canada (British Columbia)	Luxembourg
Canada (Ontario)	Netherlands
Chile	New Zealand
England	Nigeria
Finland	Scotland
France	Swaziland
Hong Kong	Sweden
Hungary	Thailand
Irish Republic	United States

The Second International Study of Mathematics is a project of the International Association for the Evaluation of Educational Achievement (IEA). IEA is an association of educational research organizations and ministries of education whose primary goals are to conduct educational research on an international level and to assist member-states in undertaking co-operative research projects. IEA has conducted international surveys in the past, including the first mathematics study (Husén, 1967) and the Six Subject Survey (Peaker, 1975; Walker, 1976). Twelve countries participated in the first mathematics study, but Canada was not among them.

British Columbia's participation in the Second International Study of Mathematics was sponsored by the Learning Assessment Branch of the B. C. Ministry of Education. The project was undertaken as an adjunct to the 1981 Mathematics Assessment which was conducted during the same school year. Participation in the international study provided an opportunity to acquire important information about the teaching of mathematics and about students' performance which did not fall within the terms of reference of the Mathematics Assessment.

1.1 Purpose of the Study

The international study had a three-fold purpose:

- to investigate the nature of the mathematics being taught in those countries participating in the study,
- to compare the ways in which mathematics was taught,
- to study the effects of that teaching on students' learning of and attitude toward mathematics.

The data collected in the study will enable researchers to construct an international portrait of mathematics education as it is currently being practiced in these 24 places.

In British Columbia, the international study has added an important new dimension to the Learning Assessment Program. Under the terms of reference of that program, surveys of students' achievement in mathematics and of instructional practices utilized by teachers of mathematics were conducted in 1977 and in 1981. These assessments were not designed to evaluate the success or failure of particular courses or programs, nor was any attempt made to relate teaching practices to patterns of achievement. Indeed, the program was designed in such a way as to protect the identities of individual students, teachers, and schools. In addition, the content tested in the two mathematics assessments, particularly at the Grade 8 and Grade 12 levels, was not selected from the prescribed curriculum for those grades. Excluding for the moment topics such as Computer Literacy, which are not in the curiculum at any grade level in this province but which were included in the 1981 Mathematics Assessment, the content of the Grade 8 mathematics assessment instruments was selected from the Grade 7 level or below. In the case of Grade 12, it was selected from the Grade 10 level or below. The decision to limit the content of the Grade 12 assessment in this way was necessitated by the fact that the instruments were administered to all Grade 12 students, whether they were studying mathematics at that level or not.

The international study, on the other hand, was designed to assess the state of the teaching and learning of mathematics in a number of highly specific ways at the Grade 8 and Grade 12 levels. The content to be tested was determined on the basis of suggestions and recommendations from each of the participating countries regarding content appropriate for students at those levels who were studying mathematics. The results obtained in the international study constitute a unique source of data concerning the teaching and learning of mathematics at the Grade 8 and Grade 12 levels in British

Columbia, information that would not be available otherwise.

1.2 Structure of the Study

Two populations of students were identified for investigation in the international study. The first, Population A, was defined as consisting of all those students enrolled in the grade where the majority of students have reached the age of 13 by the middle of the school year. In British Columbia this corresponds to Grade 8. The international definition for the second group, Population B, was intended to encompass all students in the last year of secondary school who were studying mathematics as a significant part of an academic program. In other words, students who were studying college-preparatory mathematics in Grade 12 in British Columbia were to be included in Population B, but those who were not taking any mathematics or who were taking a terminal course were to be excluded. Population B was therefore defined to consist of all students in the province who were taking Algebra 12. This included approximately 40% of the Grade 12 population. (Robitaille, 1981)

The two populations were selected as being representative of their school systems at significant junctures. At the Population A level in most countries, all students are still studying mathematics and, generally speaking, relatively little streaming or tracking has taken place. Moreover, this tends to be the highest level at which all of the students are following a common program. A comparison of some aspects of the relative structure of Population A in several countries is given in Table 1-1. Unfortunately, at the time of the writing of this report, a number of the countries had not yet supplied all of the data requested. These instances of missing data are indicated by dashes in the table.

Table 1-1
Comparative Structure of Population A

Participant	% of age cohort at grade level	% of grade cohort in Population A	% of class time for mathematics
British Columbia	100	100	14
Ontario	100	100	11
United States	100	100	12
England	-	100	13
Finland	100	-	10
Scotland	100	-	14-16
Japan	100	100	11
Belgium (Flemish)	-	78	13

Results show that, in all but one case, 100% of the age cohort at the Population A level is still in school, and that all of those students are studying mathematics. The table also shows that, in these countries, an average of about 12% of class time is devoted to the study of mathematics at this level. The average for British Columbia is somewhat higher.

An examination of the number of actual hours of instruction in mathematics at the Population A level provides some interesting additional information. In British Columbia, Grade 8 students receive an average of 133 hours of instruction in their Mathematics 8 course. This is somewhat more than students receive in Finland, where the average is between 76 and 114 hours. The B. C. average is about the same as the U. S., 143 hours, and about the same as the Japanese which is between 106 and 158 hours. In Flemish Belgium, however, Population A students receive between 210 and 245 hours of instruction in mathematics.

Population B was designed to include only those students who have taken the highest level of mathematics available in their secondary school system and who have obtained, as a result, the mathematical background required to specialize in the study of mathematics or science at the post-secondary level if they so choose.

The age-grade composition of Population B varies considerably from country to country and, as is shown in Table 1-2, so does the proportion of the grade cohort included in the population, and the proportion of class time devoted to the study of mathematics.

Table 1-2
Comparative Structure of Population B

Participant	% of age cohort at grade level	% of grade cohort in Population B	% of class time for mathematics
British Columbia	82	38	14
Ontario (Grade 12)	74	41	-
(Grade 13)	36	62	-
United States	85	12	20
England	-	30	29-57
Finland	63	41	15
France	-	28	-
Japan	87	14	11
New Zealand	16	67	40
Scotland	-	4	19

The table shows that, in British Columbia, Population B consists of 38% of the 82% of the age cohort still in school: that is, 31% of the age cohort. The corresponding rates for some of the other participants are 11% in New Zealand, 10% in the United States, 26% in Finland, and 12% in Japan. These differences in the proportion of the age cohort enrolled in Population B are undoubtedly due to the fact that, in many places, "streaming" or "tracking" programs are utilized to group students at the Population B level by achievement or ability in mathematics, and that different programs are offered for each stream. No such programs are defined in the Curriculum Guide for mathematics in British Columbia. (Curriculum Development Branch, 1978)

There is a wide variation among countries at the Population B level both as regards the proportion of class time allocated to the study of mathematics, as well as in the actual number of class hours of instruction in mathematics. In terms of percent of class time, British Columbia and Japan are among the lowest, while England and New Zealand are considerably higher. Students in British Columbia spend an average of 133 class hours on Algebra 12, and this is approximately the same as students in the United States, 160 hours, Finland, 114-171 hours, and Japan, 105-158. However, these averages are significantly lower than those to be found in Scotland, 275 hours, Luxembourg, 228-296 hours, and especially in Flemish Belgium where the average exceeds 400 hours of mathematics instruction for students in Population B.

1.3 Framework for the Study

The international study is a broadly-based, comparative investigation of the mathematics curriculum as prescribed, as taught, and as learned. For the purposes of the study, the mathematics curriculum may be viewed as consisting of three components or dimensions: the intended curiculum, the implemented curriculum, and the attained curriculum. A view of the framework for the study from this perspective is shown in Figure 1-1.

The intended curriculum is the mathematics curriculum as defined by the Ministry of Education and as codified in the curriculum guide. The implemented curriculum is the curriculum as taught by teachers in their classrooms, taking into account their personal preferences and biases as well as the composition of their class. The attained curriculum is the curriculum assimilated by the students as evidenced by their knowledge of and attitudes toward the subject.

In a paper prepared for the Ministry of Education (Robitaille and Dirks, 1982), it was shown that significant

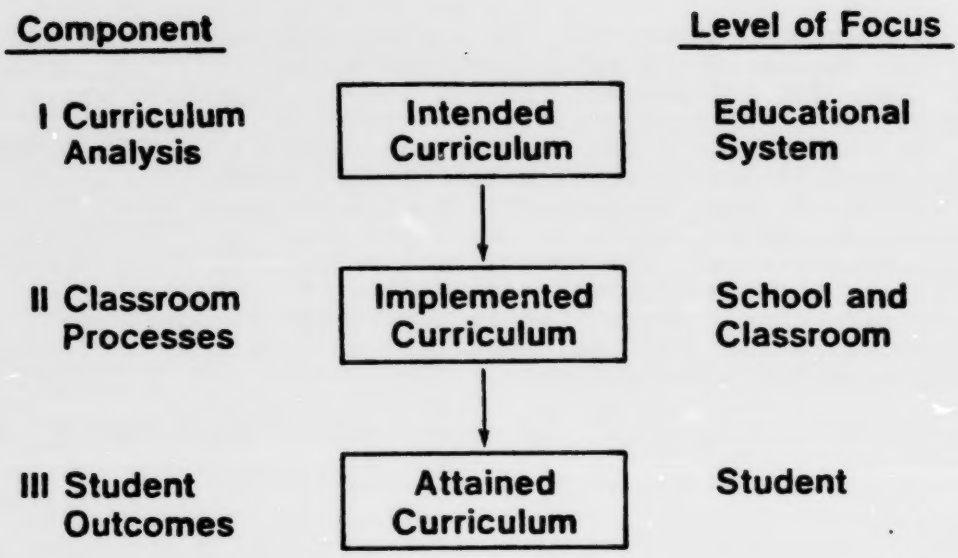


Figure 1-1. Framework for the international study.

discrepancies exist among these three components of the mathematics curriculum, both in British Columbia and elsewhere. Teachers do not follow curriculum guides slavishly. They make decisions about appropriate content for their students utilizing the curriculum guide as one of a number of sources of information and assistance. At the level of the attained curriculum, students do not learn all of the material presented to them, nor do they necessarily learn what their teachers expect them to learn. Thus, a teacher's personal view of the nature, place, and importance of mathematics might not be shared by his students who might tend to see the subject as it affects their career choices and chances for academic success rather than as a domain of interest for its own sake.

In the international study, all three dimensions of the mathematics curriculum were investigated. An analysis of the intended curriculum was conducted by examining curriculum guides and related documents from each participating country. In addition, several lengthy questionnaires were completed in each country, and all of this information was utilized to construct a composite view of the intended curriculum. The results of the international analysis of that data will be published in a separate volume, as will the international fin-

dings regarding the implemented and the attained curricula.[1]

For the study of the implemented curriculum, a number of questionnaires to be completed by teachers were developed. These included the Classroom Process questionnaires, a series of unique instruments designed to collect highly specific information from teachers regarding the methods they used in teaching specific topics in the curriculum. In this report, the data from those questionnaires has been used to produce a description of mathematics teaching in British Columbia at the secondary school level. In subsequent analyses of the data and in the international report, the results from the Classroom Process questionnaires will also be linked to student achievement data at the classroom level in an effort to examine the relationship between methodology and achievement.

In order to assess student outcomes, that is the attained curriculum, test items, questionnaires, and attitude scales were developed. The achievement items were designed to measure students' knowledge of several areas of the mathematics curriculum which had been designated as important by the organizing committees in most countries. This procedure is described in more detail in Chapter 2.

Each participant in the study had its own set of reasons for participating, and its own set of goals to be realized. For example, Australia was interested primarily in replicating the 1964 survey conducted by IEA using the same set of instruments. On the other hand, because of the Learning Assessment Program, British Columbia had no particular need for additional information on students' achievement. Instead, B. C. interest focussed on the curriculum analysis and teaching process aspects of the study.

As the study progressed these divergent interests led to the division of the project, at the Population A level, into two distinct but highly inter-related components. The first of these, the longitudinal study, utilized a pretest-posttest design on student achievement and the Classroom Process questionnaires to attempt to assess the relationship between the methodology utilized by teachers of mathematics and the achievement of their students. British Columbia was one of the participants in the longitudinal study. The second, the cross-sectional study, used a posttest-only design. Teachers in those countries participating in the cross-sectional study did not complete the Classroom Process questionnaires, and no at-

---

[1]The Curriculum Analysis volume of the Second International Mathematics Study is to be published in 1983.

tempt was made to asssess growth in students' achievement. All participants, whether in the longitudinal or the cross-sectional study, took part in the curriculum analysis component of the study. At the Population B level, no such division occurred although teachers in the United States, Ontario, and British Columbia did complete a number of Classroom Process questionnaires which had been prepared for the Population B level.

1.4 Rationale for B. C. Participation

Participation in the Second International Mathematics Study involved a significant contribution of time and effort on the part of some 200 teachers of mathematics in British Columbia and their students. In addition, a great deal of time, effort, and money were devoted to the preparation, printing, and distribution of the more than 30 data-collection instruments used in the study and to their subsequent analysis. However, the benefits to be derived from such participation were deemed to be of sufficient importance to justify the costs of participating in the project.

Foremost among the reasons for B. C. participation was the forthcoming revision of the mathematics curriculum in the province. The information to be obtained from the curriculum analysis phase of the project on an international level, as well as the local results published in this report, should be of considerable importance to the process of curriculum revision. This information will make it possible to examine the B. C. curriculum in an international context, thereby enabling the curriculum revision committee to make decisions on the future direction of the mathematics curriculum in this province on a broader base of information. It was also felt that the nature of the information to be collected in the study would be valuable, and that its value would be enhanced by participating in the study rather than merely reading the results obtained in other countries.

1.5 The B. C. Sample

The structure of the international study afforded countries a fairly wide degree of latitude in matters affecting the size and composition of the sample of classes and teachers selected, in the period of time during which data collection instruments were administered, and in the numbers of instruments that were actually used in each country. Insofar as the sampling was concerned, a major source of between-country variation in the size and composition of the sample selected was the level of analysis in which each country was interested.

Since the major objective for B. C. participation was to obtain data describing the teaching and learning of mathematics at the provincial level, and not to draw between- or within-school comparisons, a sample size of approximately 100 Grade 8 and 100 Grade 12 classes was established. The following description is a summary of the B. C. sampling plan which was approved by the IEA Sampling Referee[2] for the Second International Mathematics Study.

For the B. C. sample, Population A was defined to include all students enrolled in regular Grade 8 classes in the public schools of the province as of September 1980. Excluded by this definition were the approximately 5% of the age cohort enrolled in independent schools as well as those following programs where the level of material covered was significantly below that prescribed for the Mathematics 8 course. Population B was defined to include all students enrolled in Algebra 12 classes in the public schools as of September 1980. This excluded classes which began the course in the second semester.

In order to achieve a sample size of approximately 100 Grade 8 and 100 Algebra 12 classes stratified according to geographic zone of the province and by school size, initial samples of 125 classes at each level were drawn. In most cases this resulted in the selection of no more than one class per school. Of the 125 classes, 105 were selected for initial contact and the remainder reserved to be used as needed. In the spring of 1980, letters were sent from the Ministry of Education to all of the principals of the schools selected, soliciting their cooperation in the study and asking them to select a Mathematics 8 or Algebra 12 teacher or teachers at random from among the teachers available. In cases where it was not possible to make a random selection, the principals were asked to exercise their best judgment about which teacher or teachers to select.

Return Rates

After the sample had been drawn and the materials for the study distributed to the schools, a problem developed in the Grade 8 sample which resulted in the loss of 12 of the 105 Mathematics 8 classes in the sample. In the schools in which

---

[2]The International Sampling Committee for the Second International Mathematics Study was chaired by Dr. Malcolm Rosier of the Australian Council for Educational Research. The B. C. sampling plan was originally drawn up by Todd Rogers of U. B. C. and later revised by Tom O'Shea of Simon Fraser University and Mary Cooper of B. C. Research.

these 12 classes were located, all of the Mathematics 8 classes established at the beginning of the school year were disbanded at the end of the first semester, and new classes were set up for the second half of the Mathematics 8 course. Since the students did not stay together but were distributed among several classes and teachers, it was not possible to include them in the study. Unfortunately, this problem was not identified until it was too late in the course of the study to obtain replacements for those classes.

A summary of the return rates for both the Mathematics 8 and Algebra 12 samples is presented in Table 1-3.

Table 1-3
Summary of Returns

	Mathematics 8 (Population A)	Algebra 12 (Population B)
Number of classes selected	105	105
Complete Returns	79 (75%)	94 (90%)
Partial Returns	11 (11%)	3 (3%)
Non-participants	15 (14%)	8 (8%)

There were several reasons for the incomplete returns as well as for the non-participation of certain classes. In addition to the organizational problem already mentioned, labor disputes by support staff in a number of school districts, a strike by Canadian postal workers, teacher illness, and teachers transferring schools or school districts accounted for almost all of the remaining problems. In only a very few cases did teachers decide not to complete one or more of the instruments because of an unwillingness to cooperate further with the project.

Representativeness of the Achieved Samples

An analysis of the sample of classes and teachers from which complete returns had been obtained was conducted in order to ascertain how representative the achieved sample was of the design sample, and hence of the province as a whole. The samples had been drawn initially using geographic zone of the province and school size as stratification variables.

Population A. At the Population A level, although a substantial number of classes had been lost, those losses were spread across the six geographic zones of the province in such

a way that, in the achieved sample, each zone was appropriately represented. The data relative to this matter are presented in Table 1-4.

Table 1-4
Population A
Geographical Distribution of the Sample

Zone Number	Design Sample No. of Classes	Design Sample Percent	Achieved Sample No. of Classes	Achieved Sample Percent
1	16	15	13	16
2	39	37	25	32
3	10	10	10	13
4	19	18	13	16
5	7	7	7	9
6	13	13	11	14
TOTAL	104	100	79	100

Data were not available to enable comparisons to be made directly between the sample of Grade 8 teachers of mathematics who participated in the international study and the population of Mathematics 8 teachers. However, data on a representative sample of teachers of mathematics from Grades 8 to 12 were available from the 1981 Mathematics Assessment, and these were used as a basis for comparison.

Teachers selected to participate in the international study at the Population A level were more likely to be mathematics specialists and were more experienced than the general population of teachers of secondary mathematics. Teachers of Mathematics 8 who were in the IEA sample had an average of 14 years of teaching experience, compared to 9 years for the population of teachers of secondary mathematics. There were also indications that the IEA teachers spent a greater proportion of their teaching time conducting mathematics classes than did the population of teachers of secondary mathematics. This latter finding is not surprising since principals were asked to select a teacher of mathematics to participate in the study. It is unlikely, in such circumstances, that they would have considered selecting a teacher who taught several subject areas, or whose specialization was in a field other than mathematics.

At the student level, results show that the Mathematics 8 sample was composed of almost equal number of boys and girls, the same as was reported for this grade level in the 1981

Mathematics Assessment. The median age of students in the IEA classes was 13.8 years, the same as in the Grade 8 population as a whole. Also, 87% of the IEA teachers categorized the Mathematics 8 curriculum as they presented it to their IEA classes as "typical" rather than as "remedial" or "enriched", and 65% rated their IEA classes as "about the same" in mathematics ability as other Grade 8 mathematics classes. About 30% rated their classes as being of higher ability.

Population B. Rates of complete returns at the Population B level were extremely high. Of the 105 classes in the sample, complete returns were obtained from 90%. As is shown in Table 1-5, the returns were well distributed among the six geographic zones of the province.

Table 1-5
Population B
Geographic Distribution of the Sample

Zone Number	Design Sample		Achieved Sample	
	No. of Classes	Percent	No. of Classes	Percent
1	15	14	15	16
2	46	44	42	45
3	8	8	6	6
4	19	18	15	16
5	7	7	7	7
6	10	10	9	10
TOTAL	105	101	94	100

Teachers of IEA classes at the Grade 12 level were more experienced than the population of teachers of mathematics at the senior secondary level: that is, Grades 11 and 12. IEA teachers had an average of 16 years experience compared to 14 for the senior secondary population as a whole. In addition, the IEA teachers were more likely to be specialists in mathematics than the population. They spent an average of 73% of their teaching time in mathematics classes, whereas the population as a whole reported a figure of 54%. This means, in fact, that the IEA teachers taught one more mathematics class per day than the average.

An analysis of the sample of students in IEA classes at the Grade 12 level by gender shows that 60% were male and 40%, female. This compares with figures of 58% male and 42% female reported in the 1981 Assessment. The median age of all Grade 12 students in the province was 17.8 years in the spring of

1981, while the median age in IEA classes was 17.7. Insofar as the subject matter taught in the IEA classes was concerned, 77% of the teachers in the sample described the Algebra 12 course they taught as "typical" rather than "remedial" or "enriched", and 43% rated their IEA classes as being about equal in ability to other Grade 12 mathematics classes in the school, a designation that is, unfortunately, more than a little ambiguous since it could include students from several courses other than the Algebra 12 course. Of the remaining teachers, 12% said that theirs was the only Grade 12 mathematics class in the school, and 34% stated that their IEA class was of higher ability than other Grade 12 mathematics classes.

Summary. On the whole, it appears that, at both levels, the geographic distribution of classes participating in the international study is sufficiently close to that of the design sample to be representative of the province as a whole. Moreover, the gender and age statistics for the students in the two samples compare favorably with those for the populations. In the case of the teachers selected for participation in the study, there are some significant differences between them and the population of teachers of mathematics. In particular, the IEA teachers are more experienced and more likely to have specialized in the teaching of mathematics than their colleagues. Put another way, few teachers of subjects other than mathematics who teach one or more mathematics classes in order to complete their teaching schedules were selected for participation, neither were inexperienced teachers. In light of the purposes of the study, neither of these decisions appears unreasonable.

References

Curriculum Development Branch. Mathematics Curriculum Guide Years One to Twelve. Victoria, B. C.: Ministry of Education, 1978.

Husén, T. (Ed.). International Study of Achievement in Mathematics. Stockholm: Almqvist & Wiksell, 1967.

Peaker, G. F. An Empirical Study of Education in Twenty-One Countries: A Technical Report. Stockholm: Almqvist & Wiksell, 1975.

Robitaille, D. F. (Ed.). The 1981 Mathematics Assessment: General Report. Victoria: Ministry of Education, 1981.

Robitaille, D. F. and Dirks, M. K. Curriculum models in mathematics. In D. Robitaille (Ed.) The 1981 Mathematics Assessment: General Report. Victoria: Ministry of Education.

Walker, D. A. The IEA Six Subject Survey: An Empirical Study of Education in Twenty-One Countries. Stockholm: Almqvist & Wiksell, 1976.

CHAPTER 2

THE MATHEMATICS CURRICULUM

In the first phase of the Second International Mathematics Study, the information required to conduct a comparative analysis of mathematics curricula in the participating countries was collected. Participants were asked to submit documentation of their curricula, to complete a number of questionnaires dealing with the make-up of their curricula, and to assist in the development of the international item grids which were used as guides in determining the mathematics content to be included on the test items developed for use in the study. Since the international analysis of the data concerning the mathematics curriculum has yet to be completed, the results reported in this chapter should be regarded as tentative and preliminary. They do represent, however, the best information available at the time of publication of this report.

2.1 Development of the International Item Grids

It was the objective of the International Mathematics Committee of IEA which organized the Second International Mathematics Study to assemble a collection of test items for each of Populations A and B which would:

- constitute an adequate sample of the mathematics curriculum of each participating country;
- insofar as possible, be equally valid in each of the countries;
- enable the objectives of the study to be achieved.

Given the wide variation that exists among countries, a number of compromises had to be reached in the course of developing the items grids and the test items themselves. In addition, constraints imposed on the study by goals such as the desire to assess the extent of change in Grade 8 students' achievement over the course of the school year in which the study was conducted also influenced the composition of the collection of test items.

By means of a questionnaire distributed to participating countries early in the project, the information needed to construct content-by-behavior item grids for both populations was collected. Initially, the content dimension of the Population A item grid contained 133 categories divided among five broad areas: Arithmetic, Algebra, Geometry, Probability and Statistics, and Measurement. For Population B there were 150 categories partitioned among nine major subdivisions: Sets, Relations, and Functions; Number Systems; Algebra; Geometry; Analysis; Probability and Statistics; Finite Mathematics; Computer Science; Logic. Corresponding to each category of content, information was solicited from participants regarding the levels of cognitive behavior expected of students. Four levels of cognitive behavior adapted from Wilson (1971) were utilized: Computation, Comprehension, Application, and Analysis.

The initial versions of the content-by-behavior grids were modified on the basis of returns from the various countries participating in the study. Each participant was asked to indicate the degree of importance and emphasis attached to each cell in the grids, and to supply samples of test items corresponding to those cells. Decisions about whether or not to retain or delete a topic in the international grids were made on the basis of:

- the universality of the topic,
- the degree of importance assigned to the topic internationally,
- the degree of importance accorded the cognitive behavior level corresponding to the topic,
- the potential importance likely to be accorded a given cell as a result of changes in the curriculum which could be foreseen,
- the relevance of a given cell to the issues being addressed in the study,
- further reactions from individual national centres.

The final forms of the grids were determined at a meeting of the IEA General Assembly in Paris in the fall of 1979. Those grids are displayed in Tables 2-1 and 2-2 for Populations A and B respectively.

Table 2-1

International Grid (Population A)

CONTENT TOPICS	BEHAVIORAL CATEGORIES: Computation	Comprehension	Application	Analysis
Arithmetic				
Natural numbers and whole numbers	V	V	V	I
Common fractions	V	V	I	I
Decimal fractions	V	V	V	I
Ratio, proportion, percentage	V	V	I	I
Number theory	I	I	-	-
Powers and exponents	I	I	-	-
Other numeration systems	-	-	-	-
Square roots	I	I	-	-
Dimensional analysis	I	I	-	-
Algebra				
Integers	V	V	I	I
Rationals	I	I	I	I
Integer exponents	S	-	-	-
Formulas and algebraic expressions	I	I	I	I
Polynomials and rational expressions	I	S	-	-
Equations and inequations (linear only)	V	I	I	S
Relations and functions	I	I	I	-
Systems of linear equations	-	-	-	-
Finite systems	-	-	-	-
Finite sets	I	I	I	-
Flowcharts and programming	-	-	-	-
Real numbers	-	-	-	-
Geometry				
Classification of plane figures	I	V	I	S
Properties of plane figures	I	V	I	I
Congruence of plane figures	I	I	I	S
Similarity of plane figures	I	I	I	S
Geometric constructions	S	S	S	-
Pythagorean trianges	S	S	S	-
Coordinates	I	I	I	S
Simple deductions	S	I	I	I
Informal transformation in geometry	I	I	I	-
Relationships between lines and planes in space	-	-	-	-
Solids (symmetry properties)	S	S	S	-
Spatial visualization and representation	-	S	S	-
Orientation (spatial)	-	S	-	-
Decomposition of figures	-	-	-	-
Transformational geometry	S	S	S	-
Probability and statistics				
Data collection	S	I	I	-
Organization of data	I	I	I	S
Representation of data	I	I	I	S
Interpretation of data (mean, median, mode)	I	I	I	-
Combinatorics	-	-	-	-
Outcomes, sample spaces and events	S	-	-	-
Counting of sets, (P(A$\cup$B), P(A$\cap$B), independent events	-	-	-	-
Mutually exclusive events	-	-	-	-
Complementary events	-	-	-	-
Measurement				
Standard units of measure	V	V	V	-
Estimation	I	I	I	-
Approximation	I	I	I	-
Determination of measures: areas, volumes, etc.	V	V	I	I

Table 2-2

International Grid (Population B)

CONTENT TOPICS	BEHAVIORAL CATEGORIES: Computation	Comprehension	Application	Analysis
Sets, relations and functions				
Set notation	I	I	-	-
Set operations (e.g., union, inclusion)	I	I	-	-
Relations	-	-	-	-
Functions	V	V	V	I
Infinite sets, rationals, and reals	-	-	-	-
Number systems				
Common laws for number systems	I	I	I	-
Natural numbers	I	I	I	I
Decimals	I	I	I	I
Real numbers	I	I	I	-
Complex numbers	V	I	I	I
Algebra				
Polynomials (over $\mathbb{R}$)	V	V	V	I
Quotients of polynomials	I	I	I	-
Roots and radicals	V	V	I	-
Equations and inequalities	V	V	V	I
System of equations and inequalities	V	V	V	I
Matrices	S	S	S	S
Groups, rings and fields	-	-	-	-
Geometry				
Euclidean (synthetic) geometry	I	I	-	-
Affine and projective geometry in the plane	-	-	-	-
Analytic (coordinate) geometry in the plane	I	I	V	I
Three-dimensional coordinate geometry	-	-	-	-
Vector methods	I	I	I	I
Trigonometry	V	V	V	I
Finite geometries	-	-	-	-
Elements of topology	-	-	-	-
Analysis				
Elementary functions	V	V	V	V
Properties of functions	V	V	V	I
Limits and continuity	I	I	I	-
Differentiation	V	V	I	I
Applications of the derivative	V	V	V	I
Integration	V	V	V	I
Techniques of integration	V	V	I	I
Applications of integration	V	V	V	I
Differential equations	S	S	S	S
Sequences and series of functions	-	-	-	-
Probability and statistics				
Probability	V	V	V	-
Statistics	I	I	I	-
Distributions	I	I	I	-
Statistical inference	S	S	-	-
Bivariate statistics	-	-	-	-
Finite mathematics				
Combinatorics	I	I	I	-
Computer science	S	S	I	-
Logic	-	-	-	-

2.2 Curricular Comparisons

The item grids must be interpreted cautiously, particularly the cognitive behavior dimension. The definitions of the categories are far from being precise or prescriptive and, in spite of the fact that countries were given examples of each cognitive behavior level and that they were asked to provide sample items corresponding to each cell, there was undoubtedly considerable variation in their interpretations of them. The accuracy of the process of categorizing the level of cognitive behavior required to solve a particular item is also influenced by the prior learning and experience of individual students. Thus, an item might correctly be considered as requiring analysis level behavior from students who had never been exposed to the material before, and as requiring computation or comprehension level behavior from those students who had seen similar exercises before.

Population A

The international grid for Population A which is shown in Table 2-1 contains the importance ratings of each topic at the four cognitive behavior levels. The letter V means that the topic was rated as Very Important internationally; I means Important; and S means Important in some countries. A dash indicates that the topic was not considered to be sufficiently important to warrant inclusion of items in the pool. These ratings were used to determine the weights to be attached to to the various cells in the item pool.

In the majority of cases, the British Columbia ratings of the topics listed for each domain were lower than those reported in the international grid. The only topics which were rated as being more important for the B. C. Mathematics 8 curriculum than in the international grid were:

- number theory,
- powers and exponents,
- geometric constructions,
- Pythagorean triangles.

No topic for Population A was rated Very Important or Important at the analysis level of cognitive behavior in British Columbia.

There were two domains in which the British Columbia ratings differed rather dramatically from the international

grid: Geometry, and Probability and Statistics. In the latter case, none of the topics was rated as Very Important or Important for the Mathematics 8 curriculum. In the case of Geometry, the last eight topics were rated as being unimportant for our curriculum at this grade level.

It should be noted, however, that none of the topics just mentioned was given a rating of V or I in the international grid. They appear to be topics that are emphasized in a few countries but not in others. For example, Scotland and France appear to attach considerably more importance to these topics in geometry than do most of the remaining participants. Similarly, Scotland attaches considerable importance to the topics listed in the Probabilty and Statistics domain.

At the domain level, information derived from the importance ratings which went into the construction of the international grids may be analyzed by comparing the ratings given by different countries to each of the domains. In Table 2-3 below, comparative ratings are given for British Columbia (BC), Ontario (ONT), the United States (USA), Scotland (SCO), Japan (JPN), and France (FRA), as well as for the entire group of participants (INT) at the Population A level. The scores were calculated by assigning a value of 3 points to each V given by a participant to a cell, 2 points for each I, and, in the case of the international average, 1 for each S. When scores are compared, it should be borne in mind that these are ratings of importance. A low score in comparison to others means that an area is considered as being important but perhaps not as important as some others. Only a score of zero shows that a topic is seen as not being important. For each of the five domains, the maximum importance score is 12.

Table 2-3
Selected Comparisons of Importance Ratings
Population A

Topic	BC	ONT	USA	SCO	JPN	FRA	INT
Arithmetic	4.9	4.4	6.6	8.1	6.4	4.4	6.4
Algebra	2.3	2.8	3.1	4.7	6.0	4.2	4.3
Geometry	2.0	3.6	3.1	5.7	5.7	3.9	4.5
Probability & Statistics	0	2.4	2.1	7.0	4.2	0	3.0
Measurement	3.8	6.0	5.5	8.7	6.0	0.5	7.7
AVERAGE	2.6	3.8	4.1	6.8	5.7	2.6	5.2

According to these results, British Columbia assigns less importance to each of these five content domains at the Population A level than the average of all countries participating in the international study. The differences are greatest in the areas of Geometry, Probability and Statistics, and Algebra.

A technique called Cluster Analysis was used to analyze the data on importance ratings and to group the participants into five clusters on the basis of the similarities of their ratings. Those clusters are as follows:

I. Netherlands
(All topics rated highly, especially Algebra and Geometry)

II. Australia, Scotland, Hungary, Japan, New Zealand
(All topics rated at relatively high levels)

III. Belgium (Flemish), France, Israel, Ontario, Swaziland
(Moderate importance to all topics except Probability and Statistics)

IV. Sweden, Thailand, United States
(Arithmetic and Geometry are fairly important, but not Algebra and not Probability and Statistics)

V. British Columbia, Chile, Finland, Hong Kong, Ireland, Luxembourg
(Arithmetic and Algebra are somewhat important but not the rest)

These results should be interpreted cautiously since the importance ratings given in any country may be subject to a certain lack of reliability. For instance, a relatively low rating given to a topic in British Columbia may be due, at least in part, to a tendency on the part of the raters to understate the facts or to be rather conservative in their judgments. The fact that so many of the British Columbia ratings were lower than the international average would tend to support this view, particularly since little of the content prescribed for Mathematics 8 is missing from the international grid.

Population B

Of the nine domains listed in the international grid for Population B shown in Table 2-2, Algebra and Analysis were considered to be the most important. Logic, Computer Science, and most of the Sets, Relations, and Functions domain were given the lowest ratings. The result for the Geometry strand

is somewhat difficult to interpret since, in certain countries such as France and Belgium, the approach taken to geometry is so different that the breakdown into sub-topics shown in the grid may have had little relevance for their curricula. For British Columbia, it is important to note that the sub-topics in the Analysis strand are almost all calculus topics, and that none of these topics is contained in the Algebra 12 curriculum.

In only three cases were the B. C. ratings higher than those shown in the international grid. Each of these involved a topic which was rated as unimportant in the international grid but as of some importance to the Algebra 12 curriculum. The topic of relations is an example of one of these. In the majority of the remaining cases, the British Columbia ratings were lower than the international average.

The clearest differences between the international grid and the B. C. ratings occurred in the Analysis domain and in the Probabilty and Statistics domain. In the former case, the seven topics in the grid which deal with calculus were rated as being of significant importance internationally, but of no importance to the Algebra 12 curriculum. Similarly, none of the topics in the Probability and Statistics domain was rated as being of any importance in the Algebra 12 curriculum. In fact most of these topics form part of the Probability and Statistics 12 course which is currently being offered in some schools in the province.[1]

The importance ratings given by a number of participants to the major subdivisions of the grid are displayed in Table 2-4. The maximum value for any one rating is 12, so a result of 8 or more may be taken as an indication that a topic is considered to be very important, while a rating of 4 or less indicates that a topic is relatively less important.

There are substantial differences between the B. C. ratings of most strands and the international ones, with the greatest difference being in the area of Analysis. In fact, the B. C. ratings are lower than the international average on eight of the nine domains. Only in the Sets, Relations, and Functions strand do the B. C. ratings exceed the international ones.

---

[1]Statistics from the 1981 Mathematics Assessment show that only 2.7% of Grade 12 students have taken the Probability and Statistics 12 course.

Table 2-4
Selected Comparisons of Importance Ratings
Population B

Topic	BC	ONT	JPN	USA	SCO	FRA	INT
Sets, Relations & Functions	5.6	2.4	4.8	5.0	5.0	7.2	3.8
Number Systems	2.4	2.0	6.4	7.6	7.2	9.6	7.4
Algebra	3.1	3.7	6.0	8.3	9.0	11.3	7.3
Geometry	3.5	1.5	4.5	5.3	3.9	0.0	4.0
Analysis	1.8	2.0	6.2	5.5	9.3	8.4	8.6
Probability & Statistics	0	0	4.8	2.0	6.2	1.8	4.4
Finite Mathematics	4.0	0	6.0	4.0	8.0	10.0	6.0
Computer Science	0	0	4.0	4.0	11.0	0	4.0
Logic	0	2.0	8.0	4.0	4.0	4.0	0
AVERAGE	2.3	1.5	5.6	5.1	7.1	5.8	5.0

Cluster analysis of the Population B results on the importance ratings produced the following five groups:

I. New Zealand
(Very high importance for Algebra and Analysis; low on Sets, Relations, and Functions)

II. Belgium (Flemish), France, Israel, Luxembourg
(Low rating for Sets and Algebra, relatively high on Analysis)

III. British Columbia, Ontario, Thailand, Chile, Ireland
(Moderate importance on Sets and Algebra; Analysis not taught)

IV. Finland, Hong Kong, Scotland
(Algebra and Analysis relatively important; Probability and Statistics of some importance)

V. Hungary, Sweden, United States
(Similar to IV except that Algebra and Analysis are somewhat less important)

2.3 Item Development and Validation

In the process of collecting items for pilot-testing and inclusion in the item pool for the study, several factors were of major importance. These included the need to:

- sample the international grid as adequately as possible with items that were free of cultural bias;
- include a number of items repeated from the first IEA mathematics study in order to enable comparisons to be made where appropriate;
- include items from areas of particular interest to the International Mathematics Committee which was organizing the study--this included items dealing with topics such as basic computational skills, estimation, proportional thinking, metric measurement, and "new mathematics";
- include items at the Population A level which were judged as being most likely to provide evidence of growth in students' achievement for the longitudinal aspect of the study.

In most cases, the items selected for inclusion in the pool were found to have difficulty indices in the range 0.40 to 0.90 in the pilot testing, and mean discrimination indices in excess of 0.3 in most countries. For some of the items repeated from the first study, the anchor items, these criteria were not met. However, many of those anchor items were retained in the pool in order to enable those countries that had participated in the 1964 survey to draw comparisons about changes in students' achievement.

The original item pool for Population A contained 480 items, while the pool for Population B contained 300. Three rounds of pilot testing were conducted in various countries, and this led to extensive modifications of the items. Some items were altered, others were rejected, and some new items were added. In the end, the item pool for the longitudinal component of the study at the Population A level contained 180 items. The pool for the cross-sectional study at Population A contained 176 items, and the pool for Population B contained 136 items.

The structure of the two sets of items for Population A is shown in Table 2-5. The vast majority of the items in the two sets are identical; moreover, the distribution of items among the four levels of cognitive behavior is the same. The differences between the two were largely attributable to the fact that, for the purposes of the longitudinal study, it was necessary to include a number of items dealing with topics related to the Classroom Process questionnaires which were considered most likely to show growth in patterns of students' achievement over the period between the pretest and the posttest.

Table 2-5
Structure of the Item Pool for Population A

Strand	Number of Items	
	Longitudinal	Cross-Sectional
Arithmetic	62 (34%)	46 (26%)
Algebra	32 (18%)	40 (23%)
Geometry	42 (23%)	48 (27%)
Measurement	26 (14%)	24 (14%)
Probabilty & Statistics	18 (10%)	18 (10%)
TOTAL	180	176

Table 2-6 gives a breakdown of the item pool for Population B. The Analysis strand contained the greatest number of items, reflecting its importance on the international grid. Thirty of the Analysis items dealt with topics from calculus.

Table 2-6
Structure of the Item Pool for Population B

Strand	Number of Items
Sets, Relations and Functions	7 (5%)
Number Systems	16 (12%)
Algebra	29 (21%)
Geometry	29 (21%)
Analysis	44 (32%)
Probability and Statistics	7 (5%)
Finite Mathematics	4 (3%)
TOTAL	136

2.4 Appropriateness of the Item Pools

As part of the process of developing the item pools, each participating National Centre was asked to rate the appropriateness of the items being considered for inclusion in the pool. These ratings were made on a three-point scale: highly appropriate, appropriate, and inappropriate. An analysis of the comparative appropriateness of the Population A item pool for British Columbia and a selected number of other participants is shown in Table 2-7.

Table 2-7
Percent of Items Rated Appropriate or Highly Appropriate
Population A

Topic	BC	ONT	USA	SCO	FRA	JPN
Arithmetic	96	100	100	100	83	93
Algebra	82	97	49[1]	95	90	97
Geometry	47	92	---	90	57	88
Measurement	67	100	---	100	62	100
Probability & Statistics	94	100	100	100	28	100

[1]Approximately 10% of the items in this strand were not rated.

On the whole, the item pool for Population A appears, at least on this a priori basis, to be a valid set of items for Grade 8 students in British Columbia. Almost 80% of the items were judged to be either highly appropriate or acceptable. The proportion of acceptable items in the Geometry strand is low because of the inclusion in that strand of 15 items reflecting the approach taken to the study of Geometry in countries such as France and Belgium.

The corresponding breakdown of the item pool for Population B is given in Table 2-8.

Table 2-8
Percent of Items Rated Appropriate or Highly Appropriate
Population B

Topic	BC	ONT	USA	SCO	FRA	JPN
Sets, Relations and Functions	100	100	71	100	100	76
Number Systems	88	100	94	82	94	94
Algebra	100	86[1]	85[1]	93	85[1]	82[1]
Geometry	52	---	---	62	90	---
Analysis	32	100	100	93	93	100
Probability	14	100[1]	100[1]	43	---	86[1]
Finite Mathematics	25	---	---	100	100	---

[1]Approximately 10% of the items in this strand were not rated.

This analysis shows that while the items in the first three strands are appropriate to the Algebra 12 curriculum in British Columbia, many of the remaining items are not. The percentages of acceptable items in Probability and Statistics and Finite Mathematics are lowest, but it should be remembered

that taken together these two strands contain only 11 items. The Geometry and Analysis items, and particularly the items dealing with calculus, were the major sources of inappropriate items for B. C. students. An examination of the table shows, on the other hand, that the appropriateness ratings for these two strands were considerably higher in the other countries listed in the table.

References

Wilson, J. Evaluation of learning in school mathematics. In B. S. Bloom, J. T. Hastings, & G. F. Madaus (Eds.). Handbook on Formative and Summative Evaluation. New York: McGraw Hill Book Company, 1971.

CHAPTER 3

THE TEACHING OF MATHEMATICS: GRADE 8

As part of the Second International Mathematics Study, information regarding teaching practices was collected through the use of a series of detailed questionnaires completed by participating teachers. At the Grade 8 level the instruments included a General Classroom Practices questionnare and five topic-specific Classroom Process questionnaires[1] which dealt with the following five topics:

- Common and Decimal Fractions,
- Ratio, Proportion and Percent,
- Measurement,
- Geometry,
- Algebra (Integers, Formulas and Equations).

Each questionnaire was provided to teachers in a sealed envelope, and they were asked to open the envelope only after they had taught the topic in question and were ready to complete the questionnaire. They were also asked to complete the questionnaire as soon as possible after completing the teaching of the topic to the students in the class selected for participation in the study, the target class.

Each of the five Classroom Process questionnaires was designed to collect information about the following aspects of classroom practice:

- resources such as textbooks, worksheets, and games used in teaching;
- specific subtopics taught;
- interpretations of specific concepts such as π;

---

[1]A sample of one of the Classroom Process questionnaires for Population A is contained in Appendix D.

- content-specific methods and strategies such as procedures used for teaching subtraction of integers;
- factors teachers perceived as influencing their choice of specific concept interpretations, methods, and strategies;
- time allocated to an entire topic[2] and to individual subtopics;
- teachers' opinions regarding issues such as the need to justify for students the rules for multiplication of integers, or the place of calculators in teaching decimals.

In addition, items on several of the questionnaires concerned the types of problems and applications utilized by teachers, as well as the sources they commonly consulted in selecting or creating problems.

3.2 Treatment of Content Topics

On each Classroom Process questionnaire, teachers were given a list of between 11 and 24 subtopics related to the main topic of the questionnaire, and they were asked to categorize the type of treatment they had given to each subtopic. For example, on the Geometry questionnaire a list of 16 subtopics, ranging from congruent triangles to the Pythagorean Theorem, was given. Teachers were asked to classify each of the subtopics into one of the following five categories according to the treatment it had been given in the target class:

- taught as new content,
- reviewed and then extended,
- reviewed only,
- assumed as prerequisite knowledge and neither taught nor reviewed,
- not taught and not assumed as prerequisite knowledge.

---

[2]A listing of the prescribed content for the Mathematics 8 course in British Columbia is contained in Appendix G.

Few teachers chose the fourth category, assumed as prerequisite knowledge, for any subtopic. Out of the total of 74 subtopics on the five questionnaires, the fourth category was selected by fewer than 10% of the Mathematics 8 teachers on all but six subtopics, and five of these were from the Measurement questionnaire. It appears that few teachers felt that any of the subtopics were so well known to their students that they could be assumed without review; this was true even for such topics as the concept of a decimal and the concept of a ratio, which are first introduced at about the Grade 4 level. In the discussion that follows, the term "reviewed" is used to refer to the second and third categories combined, and "not taught", to the fourth and fifth categories combined.

Common and Decimal Fractions

On the Common and Decimal Fractions questionnaire, teachers were asked to categorize their treatment of twelve subtopics. The results are displayed in Table 3-1.

Table 3-1
Teachers' Treatment of Common and Decimal Fractions
(Percent)

	New Content	Review	Not Taught
FRACTIONS			
Developing the concept	2.3	86.0	11.7
Equivalent fractions	3.5	94.4	2.3
Addition and subtraction	8.1	91.9	0.0
Multiplication	9.4	90.5	0.0
Division	11.8	88.6	0.0
Ordering	15.5	70.3	14.3
DECIMALS			
Developing the concept	4.7	84.7	10.6
Fractions and decimals	25.9	74.1	0.0
Addition and subtracton	2.4	94.1	3.6
Multiplication	2.4	94.1	3.6
Division	4.7	91.8	3.6
Ordering	14.1	74.2	11.8

All of these topics were treated either as new material or as review by the vast majority of teachers. The largest proportion of teachers indicating that one of these subtopics was not taught at all was less than 15%. On the other hand, few of these subtopics were treated as new content; instead, they were reviewed or, more commonly, reviewed and extended.

Ratio, Proportion, and Percent

The topics of Ratio, Proportion and Percent were subdivided into eleven subtopics, and virtually all of the teachers reported that they had either taught or reviewed all eleven of them. In fact, no more than 6% reported that any subtopic was neither taught nor reviewed. These results are summarized in Table 3-2.

Table 3-2
Treatment of Ratio, Proportion, and Percent
(Percent)

	New Content	Review	Not Taught
The concept of ratio	42.5	53.7	3.7
The concept of proportion	56.8	47.0	4.2
Solving proportional equations	65.4	28.4	1.2
The concept of percent	17.5	81.3	1.2
Computing percents	45.0	53.7	1.2
Changing percents to fractions	34.6	63.0	2.5
Changing percents to decimals	36.2	62.5	1.2
Changing fractions to percents	37.0	61.5	2.5
Changing decimals to percents	36.2	62.5	1.2
Percents greater than 100	51.2	44.9	2.5
Percents less than 1	46.2	46.2	6.3

There was a considerable amount of variation in teachers' perceptions of how new most of these subtopics were for Grade 8 students. An exception to this was the concept of percent which 81% reported as belonging to one of the two review categories. The other ten subtopics were reported as new content by between 36% and 51% of the teachers and as review content by 28% to 63%. This contrasts sharply with the results for common and decimal fractions, where most topics were viewed as review material.

It cannot be determined from these results whether the lack of consensus among teachers regarding the newness of these subtopics for Grade 8 students reflects different patterns of instruction before Grade 8, different levels of achievement by students, or teachers' lack of familiarity with the content of the earlier grades. The list of subtopics does not coincide particularly well with the organization of the Curriculum Guide at this level, although solving percent problems is listed as a required topic for both Grades 7 and 8. The concept of ratio is listed in the Curriculum Guide as a basic idea for Grades 5 and 6. In spite of this, over 40% of

the teachers reported that they considered each of these subtopics as new material for their classes.

Measurement

Twenty-four subtopics were listed on the Measurement questionnaire. Of these, the concept of measurement, names of the metric units, and conversions within a system of measurement were clearly considered by most teachers as review material. Several subtopics such as perimeter of a polygon and area of a rectangle, although taught by almost all teachers, were considered as new by many and as review by others. Other subtopics, notably area of a parallelogram and area of a circle, were generally considered as new material.

Table 3-2
Treatment of Measurement
(Percent)

	New Content	Review	Not Taught
Concept of measurement	13.0	74.0	13.0
Names of SI units	18.2	74.0	7.8
Names of British units	1.3	20.0	78.6
Within-system conversion	22.1	72.8	5.2
Between-system conversion	3.9	14.5	81.6
Estimating measurements	7.8	61.1	31.2
Operations with measurements	14.5	50.0	35.6
Precision, accuracy and error	17.3	16.0	66.6
Concept of π	51.3	36.8	11.8
Linear measurement	23.4	67.6	9.1
Perimeter of a polygon	46.8	52.0	1.3
Circumference of a circle	63.2	31.5	5.3
Area of a triangle	64.9	33.8	1.3
Area of a rectangle	50.6	49.4	0.0
Area of a parallelogram	73.7	21.1	5.2
Area of a trapezoid	70.1	10.5	19.5
Area of a circle	72.7	20.8	6.5
Surface area of a rect. solid	66.2	9.1	24.7
Surface area of a cylinder	47.4	6.6	46.1
Surface area of a sphere	28.9	6.6	64.5
Volume of a rectangular solid	63.6	22.1	14.3
Volume of a cylinder and prism	61.8	6.6	31.6
Volume of a sphere	39.5	2.6	57.9
Volume of a cone and pyramid	39.5	3.9	56.6

The results show that a surprising number of these teachers teach their students British units of measurement.

The British or conventional system is not universally taught, but about 21% of the teachers did report giving it some coverage. In addition, some 18% of teachers also reported that they taught students to convert between the British and metric systems. While 29% of teachers assume that their students have some prior familiarity with the British system, only 11% expect that students entering Grade 8 will already be able to convert between systems.

The seven subtopics dealing with surface area and volume of geometric solids are definitely not viewed as review material, but their inclusion in the Mathematics 8 course by teachers is far from uniform. For instance, 47% of teachers reported teaching surface area of a cylinder as new content, while 46% reported it as neither taught nor assumed as prerequisite knowledge. Similar though somewhat less extreme results were found for the surface area of a sphere and for the volume of a cylinder, a prism, a sphere, a cone, and a pyramid.

Geometry

As shown in Table 3-3, 16 subtopics were included on the Geometry questionnaire. Of these, eight were reported as being either taught or reviewed by over 80% of the teachers. The Pythagorean Theorem and congruence of geometric figures were viewed by most teachers as content new to the Mathematics 8 course, while the other six widely-taught subtopics were viewed by many as new material and by others as review. A ninth subtopic, similarity of geometric figures, was taught by 73% of teachers.

Three subtopics were reported as neither taught nor assumed as known by 90% or more of the teachers. These were vectors, proofs, and tessellations. Similarly, 75% of the teachers reported that the topic of transformation was not part of their curriculum.

Results on three of the subtopics were mixed: spatial relations, geometric solids, and coordinate geometry. For example, less than 10% of teachers felt that coordinate geometry had been taught in prior years, but of the remainder there was an almost equal division between those who taught it and those who did not.

Table 3-3
Treatment of Geometry
(Percent)

	New Content	Review Material	Not Taught
Angles	40.8	56.6	2.6
Transformations	20.0	5.3	74.7
Vectors	2.7	5.5	91.9
The Pythagorean Theorem	80.0	13.4	6.7
Triangles and their properties	60.5	39.5	0.0
Polygons and their properties	53.9	34.2	11.8
Circles and their properties	43.4	40.8	15.8
Congruence of figures	72.4	14.4	13.1
Similarity of figures	65.3	8.0	25.3
Parallel lines	54.7	40.0	5.3
Spatial relations	19.7	22.6	56.3
Geometric solids	39.2	23.0	37.8
Ruler-and-compass constructions	67.1	23.7	9.2
Proofs	6.6	3.9	89.5
Tessellations	2.7	1.4	95.9
Coordinate geometry	49.3	9.3	41.3

Algebra

As is shown in Table 3 -4, the Algebra questionnaire contained 11 subtopics: seven dealt with integers, while the remainder dealt with formulas and equations. All of the integer subtopics were taught by almost all of the Mathematics 8 teachers, and were generally perceived as new content. Five subtopics, the concept of integers and the four basic operations with integers, were reported as taught or reviewed by all teachers. Structural properties of the set of integers were omitted by 17%, the largest proportion reporting such an omission for any of these subtopics.

The topic of solving linear equations was taught as new content by virtually all teachers. Evaluation of formulas was included by 95%, although many considered it as review, rather than new content. Deriving formulas and equations was included by 64% of the teachers, but was neither taught nor assumed as known by the remainder. Solving literal equations was taught by 30%, and excluded by the rest.

Table 3-4
Treatment of Algebra
(Percent)

	New Content	Review	Not Taught
INTEGERS			
The concept of integers	75.3	24.7	0.0
Addition of integers	84.0	16.1	0.0
Subtraction of integers	86.4	13.5	0.0
Multiplication of integers	87.7	12.3	0.0
Division of integers	88.7	12.3	0.0
Structural properties of integers	58.0	24.6	17.3
Order relations	70.5	21.8	7.7
FORMULAS AND EQUATIONS			
Evaluation of formulas	64.6	30.4	5.1
Deriving formulas and equations	52.5	11.2	36.2
Solving literal equations	29.1	1.3	69.6
Solving linear equations	92.5	3.7	3.7

Summary

All of the subtopics on the Common and Decimal Fractions questionnaire and on the Ratio, Proportion and Percent questionnaire were part of the instructional program of the vast majority of teachers. For some subtopics on the latter questionnaire, there was a lack of agreement among respondents as to whether the material was new to the Grade 8 level or was review material. On the Algebra questionnaire, almost all of the topics were taught by the majority of teachers. In this case, however, the material was more often treated as new rather than review.

The Geometry and Measurement questionnaires and the Formulas and Equations part of the Algebra instrument produced rather different results. First, a number of the subtopics were generally not taught. For example, tessellations, vectors, geometric proof, and the British system of measurement were neither taught nor reviewed by most teachers. These results are not surprising since none of these topics is listed in the Curriculum Guide for Mathematics 8.

Secondly, there were several subtopics on the two questionnaires which a significant number of teachers included in their teaching of the course, while another large proportion of teachers did not, but which few teachers felt had been previously taught. Surface area and volume, spatial relations, coordinate geometry, deriving formulas, and solving literal equations are all examples of such topics. These results suggest that the grade placement and suggested treatment of these

topics should be considered carefully in the next revision of the curriculum because there is a considerable lack of agreement among teachers as to whether or not these topics should be included in the course and this may be due to a lack of clarity in the Curriculum Guide.

3.3 Concept Interpretations and Teaching Methods

The Classroom Process questionnaires contained a number of items designed to collect information on the various approaches to or interpretations of concepts used by teachers in presenting certain topics. Teachers were given lists of possible teaching approaches and were asked to rate the extent to which they employed each alternative. The response categories were:

- emphasized (used as a primary explanation or technique, referred to extensively or frequently),
- used but not emphasized,
- not used.

For example, on the Common and Decimal Fractions questionnaire there were 10 interpretations of fraction concepts listed, and teachers were asked to indicate which of these they had used with their students. Two of these interpretations are shown in Figure 3-1.

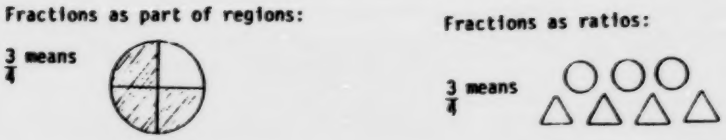


Figure 3-1. Two possible interpretations of fraction concepts.

In responding to these items, teachers could have indicated that they had emphasized all of the approaches listed, or, more likely, that one or two were emphasized, several others used but not emphasized, and the rest not used at all.

Each questionnaire also contained a number of statements such as "Rules for operations with common fractions and decimals should be memorized." Teachers were asked to indicate the extent of their agreement or disagreement with each such

statement by selecting a response from a five-point scale ranging from strongly agree to strongly disagree. For the purposes of this discussion the five categories have been reduced to three: agree, disagree, and undecided.

Decimals and Fractions

In interpreting the concept of decimal fractions every teacher in the sample reported teaching that a decimal is another way of writing a fraction, for example, 0.17 = 17/100, and that decimals are an extension of the place value system of notation. Most teachers emphasized these points in their teaching. About 90% of the teachers also reported teaching that a decimal is the coordinate of a point on the number line and that a decimal may be represented by a series: for example, 0.243 = 2/10 + 4/100 + 3/1000. Most teachers reported that they used, but did not emphasize, these latter two interpretations.

Two of the interpretations of decimals listed on the questionnaire were similar to ones customarily associated with teaching introductory concepts of fractions. In the first of these, 50% of the teachers indicated that they used decimals in part-whole situations, while, in the second, 28% said they used decimals in subset-set situations. It would appear in other words, that although teachers tell their students that fractions and decimals are simply alternate ways of expressing the same numbers, many of them do not, in fact, apply both in similar situations.

Two possible approaches to the teaching of multiplication of decimals were described in the questionnaire. While 76% of the sample indicated that they had related operations with decimals to the corresponding operations with fractions, three times as many teachers reported that they had used this technique rather than emphasized it. Even more teachers, 98% of the sample, agreed that they related operations with decimals to operations with whole numbers, and taught rules for placing the decimal point in the answer. Most teachers, in fact, emphasized this approach in their teaching.

These results may mean that multiplication of decimals is taught primarly as a rote procedure, but this assertion must be tempered with the fact that most teachers indicated some use of both techniques and that a distinction between emphasis and use could mean many things in practice. However, results on other topics surveyed in the Classroom Process questionnaires provide further support for concluding that rote approaches are used frequently in the teaching of Mathematics 8. For instance, 29% of the teachers responding to the Algebra questionnaire agreed that their teaching of the multiplication of integers involved no conceptual development at all but con-

sisted simply of the presentation of rules.

Partly beacause of the widespread and ever-increasing availability of calculators and the introduction of the metric system, it has been suggested that, after exposure to the basic notions of fractions in the primary grades, instruction should proceed to the introduction of decimal fractions and then to operations with decimals. Arithmetic operations with fractions could be delayed to later grades.

It is not clear from such a result whether or not teachers of Mathematics 8 would concur with such an approach. While 48% did agree that decimals and operations with decimals should be emphasized more than common fractions and operations with them, a significant number, 34%, disagreed. Moreover, 66% felt that instruction on common fractions should precede instruction on decimals.

All but one of the teachers agreed that computation with fractions should be taught and 71% felt that drill should continue until students have developed a high degree of skill at computing. Some 65% were of the opinion that skill at computing indicates understanding. Only 31% agreed with the statement that estimation, approximation, and checking the reasonableness of an answer were more important than becoming skilled in computing with common fractions and decimals. Almost 50% disagreed and 22% were unsure.

A similar emphasis on computational skills was apparent in the case of percent. Only 15% agreed with the opinion that computation with percent should be done with hand-held calculators, and no teacher strongly agreed. On the other hand, 66% disagreed with this statement and a quarter of these indicated strong disagreement. Thus, while most teachers of mathematics in Grade 8 expressed positive opinions about the use of calculators by students, they were adamant about the importance of developing students' computational skills.

Integers

The concept of negative numbers and the basic operations on the set of integers are important areas of new content in Mathematics 8. The number line was used by all teachers in introducing integers, and 80% reported emphasis of this approach. Teachers seldom, on the other hand, used directed segments or vectors on the number line to represent integers. Physical situations such as money, credit and debit, and time before and after, were used frequently to develop integers, as was the idea that integers are needed in order that equations such as $n + 7 = 5$ can be solved.

The number line and physical situations were also heavily used to conceptualize the addition of integers, as was the presentation of rules. Most teachers also emphasized the presentation of rules for subtraction of integers. The number line was used less frequently for this operation.

The rules for multiplication of negative integers are difficult to rationalize. Teachers most frequently reported developing those rules through the use of patterns such as the following:

$$\begin{aligned} +1 \times -3 &= -3 \\ 0 \times -3 &= 0 \\ -1 \times -3 &= +3 \\ -2 \times -3 &= +6 \end{aligned}$$

Three-quarters of the teachers found the repeated addition model of multiplication useful for integers with almost as many reporting some use of physical situations.[3]

Formulas

The study of various formulas, such as those involving the perimeter, area, and volume of simple figures, and functional relationships specified by equations is an important part of the Mathematics 8 curriculum. All teachers appear to devote a significant amount of time to the study of various formulas, but there were some significant differences of opinion among them regarding whether or not specific formulas should be taught.

The Algebra questionnaire contained descriptions of five general techniques which teachers might employ in instruction related to formulas. Four options involved student generation of formulas using physical, visual, or mathematical data while the remaining option involved a didactic approach. The didactic approach, presenting formulas and explaining the meaning of the terms in the formulas, was the method most frequently employed. Seventy-eight percent of teachers reported emphasizing this method, and an additional 14% reported some use of it. The other four techniques were each reported as emphasized by less than 25% of the sample. Two of these, providing data from which formulas or equations were developed and having students create new formulas based on known simpler formulas,

---

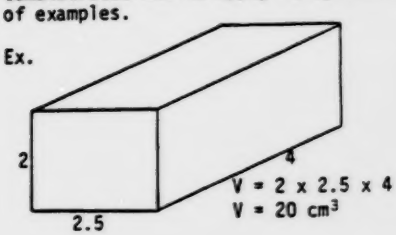
[3]As was noted elsewhere, 29% of teachers indicated no real development of the rules for multiplication of integers, they merely gave students rules.

were reported as emphasized or used by over half of the teachers. Slightly less than 50% reported having students inspect graphs and develop formulas to express the relationships portrayed in these graphs. The most concrete approach, having students collect data on related variables and formulate the relationship between the variables, was not used at all by most teachers.

Results on the Measurement questionnaire confirmed teachers' preference for didactic over experimental methods when teaching formulas. Teachers were asked which of the three options shown in Figure 3-2 they employed in teaching the formula for finding the volume of a rectangular solid.

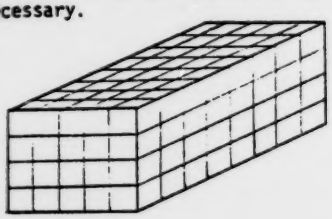
I presented the formula V = l x W x h or V = (area of base) x height and demonstrated how to apply it by means of examples.

Ex.



I presented a physical model of a right rectangular prism (box) with its faces marked off in square units, as illustrated below. I had students generate the formula by relating the number of cubic units contained in the prism to the dimensions of the box, giving hints only if necessary.

Ex.



I provided my students with unit cubes and asked them to build rectangular prisms of specified dimensions. I asked them to relate the number of unit cubes used to the given dimensions, giving hints only if necessary.

Figure 3-2. Teaching the formula for the volume of a rectangular solid.

The first method, the most didactic of the three, was emphasized by 58% of the sample and used by another 30%. The second method, emphasizing meaning and student participation, was emphasized by 36% and used by 33%. The final one, a concrete experimental method, was emphasized by 4% and used by 6%. Fully 90% of teachers made no use of this method at all. The pattern of teacher preference is both clear and striking in this case.

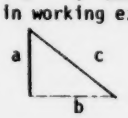
The Pythagorean Theorem

The Pythagorean Theorem is listed as new content for Mathematics 8 in the Curriculum Guide. Eighty percent of the teachers taught it as new material while 13% considered it as review; only 7% did not teach this subtopic at all.

Seven possible approaches to this topic were described in the questionnaire; two of these were used predominantly. The first was consistent with methods used for other subtopics. It involved the presentation of the formula and having students use it in exercises. Over 80% of teachers either used or emphasized this didactic approach. An almost identical pattern of use was reported for a second method in which squares are drawn. These two frequently-used approaches are shown in Figure 3-3.

Over 60% of teachers reported that they had their students measure the sides of various right triangles, record the results, and discuss the pattern that emerged. This high rate

I gave my students the formula $a^2 + b^2 = c^2$ and had them use it in working examples.



I used diagrams like the following to show that, in a right triangle $a^2 + b^2 = c^2$

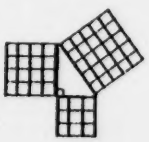


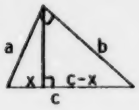
Figure 3-3. Frequently used approaches to the Pythagorean Theorem

of use of a laboratory approach involving measurement data is encouraging, and is a departure from the general orientation in Mathematics 8 teaching as reported on the questionnaires. About 40% of teachers also reported spending some time in an historical discussion of Pythagoras and Euclid with their classes.

Three of the approaches listed for the Pythagorean theorem were used very infrequently by teachers. The traditional Euclidean proof of the Pythagorean Theorem involving area was used by virtually none of the teachers, only 4% in fact. Similarly, the algebraic justification involving similar right triangles was used by only 10%. Perhaps more surprising was the low rate of utilization of the informal area argument involving the two diagrams shown in Figure 3-4 which can be illustrated on a geoboard or with models. Only 26% of teachers

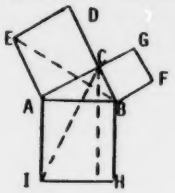
I presented a formal deductive "algebraic" argument.

Ex: Using similar right triangles, proportions can be set up to yield $a^2 + b^2 = c^2$



I presented a formal deductive argument using area.

Ex: This figure is sometimes used to present a formal proof.



I presented an informal area argument using physical, e.g. geoboards, or pictoral models.

Ex: I showed that the two squares had equal area.

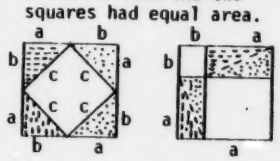


Figure 3-4. Infrequently used approaches to the Pythagorean Theorem.

reported any use of this strategy, and another 20% indicated that they did not use this approach because it was not well known to them.

Congruent triangles, parallel lines, and other topics

In categorizing their general approach to the teaching of geometry, 62% of teachers reported that they emphasized, and 19% that they used, an informal approach based on inductive reasoning, measurement, and intuition. Only 16% reported any orientation toward an axiomatic approach involving proofs of theorems, and no teacher reported emphasizing formal geometry. Responses from the opinion section of the Geometry questionnaire show that teachers neither proved theorems nor presented concepts in an order determined by an axiomatic system. While the teaching of geometry is decidedly not formal, in the sense of axiomatics, it is less clear that it is generally taught from an intuitive, inductive, or measurement base.

Although these results indicate that the teaching of geometry in Grade 8 is not particularly formal, in a mathematical sense, the data from several items do show that, the teaching style is very formal indeed. For example, the most commonly utilized method for teaching students about the concepts of congruent triangles, similar triangles, and parallel lines was to state the relevant definitions and give related

examples. This method was used or emphasized, usually emphasized, by most teachers. No information about where, in the teaching of these subtopics, the statement of definitions and properties comes is available. It is clear, however, that the statement of definitions was usually emphasized, while other strategies may have been used but were not emphasized.

For both congruent triangles and parallel lines the second most frequently emphasized approach, used to some extent by over 80% of the teachers in each case, was having students perform constructions with ruler and compass. This activity was used somewhat less frequently in studying similar triangles. Rulers and protractors were also employed frequently. As before, this was usually at the level of use rather than of emphasis. Instruments such as the set square were not used very frequently, nor were geoboards. While teachers believe that concrete models and instructional aids are essential in teaching geometry, classroom practice customarily does not seem to go beyond ruler, compass, and protractor.

Environmental examples were frequently cited in studying parallel lines and similar triangles, and much less frequently in studying congruent triangles. Transformations were used by only a few teachers in dealing with these subtopics.

The Number π

The number π is not specifically mentioned in the Curriculum Guide, but apparently it is to be introduced at least by Grade 8 since this is the first year that the formulas for the circumference and area of a circle are listed as learning outcomes. In fact, 37% of teachers taught π as a review topic, while 12% did not include it in their teaching. The remaining 51% introduced the number π as new material.

Seven possible interpretations of the number π were given in the Measurement questionnaire. The didactic approach, "I told my students that π = 22/7 or 3.14[4]", was emphasized by 62% and used by 18% of teachers, making it the most popular choice. An almost identical total of 79% discussed π as the ratio of the circumference of a circle to its diameter and told students it was an irrational number. About 55% of teachers reported that they had students measure and find the

---

[4]The statement regarding the value of π is, of course, false. The given values are merely rational-number approximations of the value of π.

ratio of the circumference to the diameter of a number of circular objects. This result is very encouraging. While it is consistent with teachers' expressed opinion that the best way to learn measurement is by measuring things, options involving such activity for other subtopics were used less often.

Three other ways of introducing π were not used at all by most teachers. These approaches dealt with estimating π using Buffon's needle problem, using a sequence of regular polygons with increasing numbers of sides to obtain better approximations of π, and defining π as the area of a unit circle and using finer and finer grids to approximate the area. The fact that these approaches were seldom used is not surprising, although the availability of computers could make them more appropriate and better known in the future.

Spatial Relations

An apparent inconsistency between the stated opinions of teachers and their methods of instruction as reported on the questionnaire occurred in the area of spatial relations. Eighty-five percent of teachers agreed with the statement that activities to improve students' ability to visualize spatial figures should be included in the instructional program. However, the eight techniques for teaching spatial relations listed on the questionnaire were reported as utilized to a relatively limited extent. For example, none of these techniques was emphasized by more than 17% of the teachers. Making a two-dimensional drawing for a given three-dimensional object was the most frequently cited method, with 57% reporting some use. Using two-dimensional patterns or nets to build three-dimensional figures, and designing two-dimensional patterns for three-dimensional objects were used by 41% and 38% respectively. Drawing plans of geometric solids and building models of intersecting planes in space were used by 24% and 11% respectively.

3.4 The Use of Instructional Time

Evidence exists to support the notion that student "time-on-task" is highly correlated with achievement. In order to obtain information about this issue, the Classroom Process questionnaires included a number of items dealing with the amount of time and emphasis given to the topics covered by these questionnaires: Common and Decimal Fractions; Ratio, Proportion and Percent; Measurement; Geometry; and Algebra.

Topic Inclusion

The first item on each Classroom Process questionnaire asked if the topic was, in fact, part of the curriculum for that class. If not, this was to be indicated and the rest of the questionnaire disregarded. Of the 415 questionnaires completed by the Grade 8 teachers, only 14 of those returned indicated that the topic dealt with in that questionnaire had not been taught. Of these, six were in Measurement; five in Geometry; two in Ratio, Proportion and Percent; and one in Common and Decimal Fractions. None of the Algebra questionnaires was returned blank. In other words, all of the general topic areas covered by the questionnaires had been dealt with, to some extent, by over 90% of the teachers.

Time Variation

While most teachers reported that they included all five of the topics dealt with in the Classroom Process questionnaires in their courses, there were substantial differences in the amount of time spent on each topic. This information is summarized in Table 3-5 which shows, for example, that 10% of the teachers allocated at most 4 periods to the teaching of Measurement. The topics of Common and Decimal Fractions and

Table 3-5
Number of Periods Allocated to Each Topic

Topic	Percentile 10th	25th	50th	75th	90th	Mean	S.D.[1]
Fractions	4	7	10	14	20	11.8	7.0
Decimals	5	7	10	14	18	10.6	5.1
Ratio	7	11	15	18	25	15.2	6.8
Geometry	10	16	21	30	38	23.4	10.0
Measurement	6	10	15	22	30	17.3	10.0
Integers	8	11	14	18	22	15.4	7.6
Equations	6	10	14	21	30	16.9	9.6

[1]Standard Deviation

Algebra have each been subdivided into two areas, so that the table contains seven, rather than five, topics. While the lengths of class periods did vary between 35 and 75 minutes, reporting the allocated time in terms of minutes rather than class periods does not provide any substantially different information. For that reason it has not been included here. In fact, the majority, 87%, of teachers reported period lengths of between 50 and 60 minutes.

Table 3-5 shows that 25% of the teachers spent seven periods or less on the topics of Common and Decimal Fractions, and that another 25% spent 14 periods or more. This 2:1 ratio is closely approximated in all topics. When the 90th and 10th percentiles are compared, the ratios become as high as 5:1. In other words, there is a considerable amount of variation in Mathematics 8 classrooms as regards the amount of time allocated to specific content areas.

3.5 The Use of Instructional Resources

The instructional resources, both textbooks and other materials which teachers utilize, are among the most significant determinants of the curriculum as it is actually taught. Each Classroom Process questionnaire contained a list of eight such materials, including the titles of the three textbooks approved for Mathematics 8, and teachers were asked to indicate whether each resource listed was used as a primary source of material, a secondary source, or was not used at all. These categories and the mean response rates across the five questionnaires are shown in Table 3-6. The usage score was computed by weighting primary sources twice as heavily as secondary ones. The range of values for this score is 0-100, with a score of 0 indicating that the source was not used by any teacher, and a score of 100 indicating that it was used as a primary source by all teachers.

Table 3-6
Use of Instructional Materials
(Percent)

	Primary Source	Secondary Source	Not Used	Usage Score
School Mathematics 2	31	29	41	45.5
Mathematics II	37	27	37	50.5
Essentials of Math. II	4	9	87	8.5
Other published textbooks	23	49	28	47.5
Locally produced materials	35	40	25	55.0
Individualized materials	5	11	85	10.5
Films, filmstrips, or models	1	12	87	7.0
Laboratory materials	1	20	79	11.0

Locally produced materials, including worksheets, were the most widely used resource. Next in order of usage score were the textbook, Mathematics II, textbooks other than the prescribed ones, and School Mathematics 2. In fact, the usage scores for all four of the top categories were approximately

equal. The textbooks Mathematics II and School Mathematics 2, were used as primary or secondary sources by 64% and 60% of teachers respectively, and this finding indicates that multiple textbooks are being used in a significant number of classrooms.

The other four categories of instructional materials had very low usage scores. Between 79% and 87% of teachers indicated that they made little or no use of any of these materials. These categories include the third prescribed text, Essentials of Mathematics II, computer assisted instruction, films, and laboratory materials.

It might be expected that the usefulness of some instructional materials, models for instance, would differ significantly among topics. The data, however, lend little support to this hypothesis, as most usage patterns were quite uniform across topics. More use of laboratory materials is reported for geometry and measurement than for the other topics, as is shown in Table 3-7, but the usage scores are still low. In analyzing the usage by topic of the two most heavily used textbooks, a noticeable shift is evident for geometry. Mathematics II is more popular than School Mathematics 2 for this topic by a fairly large margin, 25% more use it as a primary source, while this is not true for the other topics.

Table 3-7
Use of Laboratory Materials
(Percent)

	Primary Source	Secondary Source	Not Used	Usage Score
Fractions and Decimals	1	19	80	10.5
Ratio, Proportion, and Percent	0	12	88	6.0
Geometry	3	31	66	18.5
Measurement	2	24	74	14.0
Algebra	1	13	86	7.5

Teachers were also asked to supply information about the sources they consulted for applications and problems. This was done for the Algebra questionnaire, and for Ratio, Proportion and Percent. These results are presented in Table 3-8.

Prescribed textbooks are the most frequently reported source of problems and applications, with worksheets being a close second. Supplementary texts are also frequently consulted, but usually as a secondary rather than a primary

Table 3-8
Sources of Applications and Problems
(Percent)

	Used Frequently	Used Occasionally	Not Used	Usage Score
Prescribed textbooks	70	25	5	82.5
Supplementary textbooks	21	64	16	53.0
Worksheets	63	30	8	78.0
Curriculum guide	13	42	45	34.0
Published articles	0	33	67	16.5
Students' suggestions	1	46	53	24.0
Real world applications	4	55	41	31.5

source. The other four categories are not used at all by between 41% to 67% of teachers, and are seldom used as primary sources.

Finally, teachers were asked to indicate which instructional aids they utilized in teaching topics in geometry and measurement. All teachers used rulers and compasses frequently. Protractors and graph paper were used by almost all teachers but only occasionally. Of the remaining 10 types of instructional aids for geometry and measurement, three were used occasionally by many teachers: models of solids, patterns, and paper for folding. Other categories such as computers for graphics, geoboards, and films were never used by 80% or more of the teachers.

It would appear from other information obtained from teachers that, although many instructional aids and resources are neither widely nor frequently used, teachers do not feel that a lack of such materials is among their most urgent concerns in helping students learn. When they were asked to rate the importance of selected factors which might explain a lack of satisfactory student progress in mathematics, only 20% felt that having a limited number of physical resources was an important reason for lack of satisfactory progress by students. By contrast, over 85% cited students' lack of ability, indifference, and student absenteeism as being important. Moreover, two-thirds rated students' misbehavior as an important reason for unsatisfactory progress, while about 50% felt they lacked expertise in dealing with students with learning disabilities. While teachers undoubtedly want high quality instructional materials they generally agree that factors other than inadequate materials are mainly responsible for the learning problems experienced by students.

3.6 Influences on Content Decisions

Little or no information exists in British Columbia regarding the factors that influence teachers' decisions about what content to teach and how to teach it. Prescribed textbooks are certainly important influences, but there may be other influences which are also important in this regard. In order to obtain some additional data concerning this question, teachers were asked to indicate which factors exerted the greatest influence on their decision-making processes regarding content and method.

Figure 3-5 provides an example of the the kinds of items which were used for this purpose. Four interpretations of integers are shown, out of the five given on the questionnaire. The fifth involved the use of physical situations. Teachers were asked which of these interpretations they emphasized, which they used but did not emphasize, and which they did not use at all. Those teachers who reported that they had either emphasized or had not used a particular interpretation were then asked to select, from a list of eight, the two reasons which had most strongly influenced their decision.

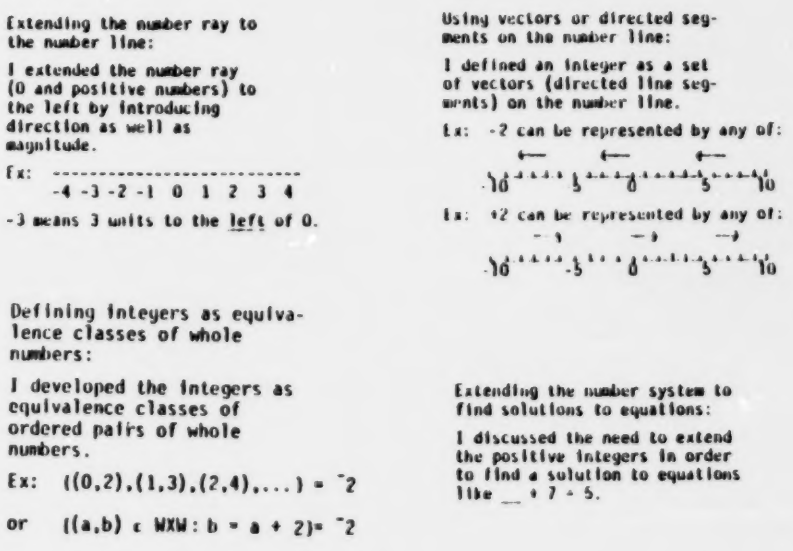

Extending the number ray to the number line:
I extended the number ray (0 and positive numbers) to the left by introducing direction as well as magnitude.
Ex:
-4 -3 -2 -1 0 1 2 3 4
-3 means 3 units to the left of 0.
Using vectors or directed segments on the number line:
I defined an integer as a set of vectors (directed line segments) on the number line.
Ex: -2 can be represented by any of:
Ex: +2 can be represented by any of:
Defining integers as equivalence classes of whole numbers:
I developed the integers as equivalence classes of ordered pairs of whole numbers.
Ex: {(0,2),(1,3),(2,4),...) = ⁻2
or {(a,b) ∈ WXW: b = a + 2}= ⁻2
Extending the number system to find solutions to equations:
I discussed the need to extend the positive integers in order to find a solution to equations like __ + 7 = 5.

Figure 3-5. Sample of content decision items.

Teachers were asked to rate the importance of decision-

making influences for the 11 content topics listed below. The numbers in parentheses indicate the number of different interpretations which the teachers were asked to evaluate. For example, on the Common and Decimal Fractions questionnaire, teachers were asked to react to 10 different possible interpretations of the concept of fractions.

- concept of a fraction (10)
- concept of decimals (6)
- concept of proportion (4)
- sum of the angles in a triangle (8)
- Pythagorean theorem (7)
- the number π (7)
- formula for the area of a parallelogram (8)
- concept of integers (5)
- adding integers (3)
- subtracting integers (6)
- solving linear equations in one variable (5)

Reasons for Emphasis

Table 3-9 contains the weighted ratings of relative influence given to each reason for emphasizing an interpretation or method on each of the five Classroom Process questionnaires. Since teachers were asked to select two responses for each item, the maximum percent possible for any category is 50. That is, a score of 50 indicates that all teachers chose that option for every interpretation or method emphasized.[5]

In the table, acronyms have been used to refer to the questionnaires. The designations are as follows:

CDF--Common and Decimal Fractions
RPP--Ratio, Proportion and Percent

---

[5]Since teachers occasionally gave only one reason for emphasizing a topic, the actual maximum percentage is somewhat over 50 and varies by topic.

ALG--Algebra
GEO--Geometry
MEA--Measurement

Table 3-9
Factors Influencing Decisions to Emphasize Approaches
(Percent)

Reason for Emphasizing	Questionnaire					
	CDF	RPP	ALG	GEO	MEA	Overall
Well-known to me	10	12	14	14	20	13.7
In a curriculum guide	9	7	5	4	7	6.1
Easy for students to understand	23	30	38	39	30	32.8
Enjoyed by students	4	6	6	10	11	7.4
Related to math of prior grades	19	15	8	5	9	11.1
Useful in math of later grades	28	23	17.	12	13	19.1
I was taught it was appropriate	3	3	2	3	4	2.8
Emphasized in student text	6	4	9	7	6	7.3

From the ranges of the response rates, it is apparent that the reasons given for emphasis vary with content; however, several generalizations can be made. The reason "easy for students to understand" was the one most frequently selected by teachers, twice as often as the second choice "useful in math of later grades". Furthermore, it was the most frequently chosen factor for 10 of the 11 topics listed earlier in this section. In the case of three of those topics, this reason made up 40% or more of the responses given. This means that, for those categories, being "easy for students to understand" was a primary reason for teachers emphasizing a particular approach or interpretation about 80% of the time. For example, teachers perceive the ease with which their students process a particular interpretation of ▪ or negative integers as an important factor in determining how to present these concepts. The opinion that an approach will be "enjoyed by students" is listed as an important factor in this process far less frequently.

As was noted earlier, the opinion that a particular interpretation or method will be useful in later grades has the second greatest influence. Interestingly, this factor achieved relatively high means largely for review content: concept of a fraction, concept of decimals, and concept of proportion.

The third most frequently chosen reason was "well known to me". This can be contrasted with a related option which was

selected by almost no one: "I was taught it was appropriate". This latter option was intended to assess the influence of pre-service and in-service education at this level of decision-making, but the wording was somewhat ambiguous. The fact that certain interpretations and strategies are chosen frequently because they are well known raises the question of how they came to be known to teachers. It would be reasonable to expect that much of this knowledge would have come from mathematics and mathematics education courses. However, the structure of these items on the questionnaires, the fact that respondents were limited to two choices, and the wording that was used prevent a firm conclusion from being drawn to the effect that teachers' knowledge of content and their training are not highly related.[6]

The fact that a particular interpretation of a concept or method was included in one of the student textbooks was apparently not an important factor in teachers' decisions to emphasize it--at least not as they perceived and reported its influence. This may be contrasted with the importance teachers attached to "not emphasized in the student text" as a reason for not emphasizing an interpretation or method. In no case did the mean response rate for "emphasized in student text" exceed 10% of all reasons given for emphasis.

The option, "in B.C. or local curriculum guide", was chosen almost as infrequently as "I was taught it was appropriate". It is perhaps not surprising that teachers attach relatively little importance to the Curriculum Guide in this regard, since it merely enumerates topics to be covered but provides no details about the the coverage or of methods and materials to be used. It may still be true that the Curriculum Guide is an important determinant of what content is to be taught in Grade 8, but this question was not addressed directly.

In summary, teachers of Mathematics 8 report being highly influenced in deciding how to teach concepts and procedures by the anticipated outcome in student understanding. They also perceive the future usefulness of a topic as being influential

---

[6]On the General Classroom Practices questionnaire teachers were asked how difficult it would be for them to teach Mathematics 8 without knowledge acquired in their university training. Over 70% felt that it would be difficult to teach without the knowledge they had gained in their mathematics courses. About 20% expressed a similar opinion concerning general education and mathematics education courses.

in their decision-making. They are less concerned with how particular interpretations and methods are related to approaches used in prior grades, but this factor is of substantial importance for areas of review such as decimals and fractions. Teachers frequently report approaching subtopics in a particular way because it is well known to them, but the source of this knowledge is not clear since they seldom indicate such potential sources as texts, training or curriculum guides as being influential at this level of decision-making.

Reasons for Non-Use

In those instances where teachers indicated that they did not use a particular concept interpretation or strategy in their teaching, they were asked to specify the two most important reasons for this decision. The eight options they selected from were parallel in form to the eight listed for those interpretations or strategies they did use. There were both similarities and differences between the ways in which teachers responded to the reasons listed for emphasizing a concept or interpretation and to the reasons given for not using them. The overall results for the five questionnaires are summarized in Table 3-10.

Table 3-10
Factors Influencing Decisions Not to Use Approaches
(Percent)

Reason for Non-Use	Questionnaire					
	CDF	RPP	ALG	GEO	MEA	Overall
Not well-known to me	13	13	15	18	20	16.8
Not in curriculum guide	8	10	9	10	10	9.4
Hard for students to understand	13	24	31	27	22	24.6
Disliked by students	8	7	8	5	4	6.1
Unrelated to previous math	5	2	7	7	5	6.1
Not useful in later grades	16	10	8	5	6	7.8
I was taught it was inappropriate	2	2	3	2	1	2.0
Not emphasized in the text	35	31	20	25	32	27.2

"Not emphasized in the text" was the most frequently selected reason for non-use; it accounted for 27% of the options selected across all topics. It was also the most frequently chosen factor for 6 of the 11 individual topics listed earlier in this section. This means that, while teachers only occasionally felt that they emphasized an inter-

pretation or strategy because of its presence in the student text, they frequently indicated that an approach was not utilized because of its absence from the text. In other words, when a strategy or interpretation is covered in the textbook, many teachers are reluctant to interpret it in ways that differ from the treatment given in the textbook.

Teachers rated "hard for students to understand" as the second most important reason for not using a given interpretation or strategy. Just as teachers emphasized some approaches because they perceived that students would find them easy, they did not use others because they felt that students would find them difficult.

"Not well known to me" was the third most influential factor in teachers' decisions not to use an approach to content. It accounted for 17% of the reasons selected. This particular reason was highly dependent on the item being discussed. For instance, only two teachers indicated using Buffon's needle problem in teaching the concept of π, with most teachers indicating that this approach was not well-known to them. On the other hand, while almost 50% of the teachers did not have their students actually measure the circumference and diameter of circular objects in teaching π, only four stated that such a method was not well known to them.

None of the remaining five reasons was selected at a consistently high level across topics. This was a departure from the pattern of parallel reasons given for emphasis only in the case of "not useful in math of later grades". This option accounted for only 8% of those selected, while its counterpart "useful in math of later grades" accounted for 19%. Thus, while future utility appeared to have a fairly strong influence on decisions to emphasize approaches to content, a perceived lack of future usefulness was not generally seen as a good reason for not using an interpretation or strategy.

Two factors appeared to weigh heavily in teachers' decisions about which teaching approach to emphasize. Approaches which were not in the textbook were not be used by teachers, and neither were approaches which were perceived as being difficult for students to understand.

3.7 Variety of Interpretations and Strategies

Within each of the topics discussed in the previous section, teachers were asked to react to between three and ten possible interpretations or methods of instruction. This information was used to investigate the degree of variety of approaches used by teachers. That is, the data were used to investigate the number of different approaches utilized by teachers in teaching each of these topics.

For each of the eleven topics, with the exception of addition of integers which had only three suggested approaches, a frequency distribution of the approaches used or emphasized was constructed. This information is summarized in Table 3-11.

Table 3-11
Variety of Approaches Emphasized
(Percent)

Topic	Number of Approaches 0-1	2-3	4 or more	Total Items
Interpretations of fractions	24	57	20	10
Interpretations of decimals	32	60	8	6
Interpretation of proportion	54	39	8	4
Angle sum in a triangle	72	23	7	8
The Pythagorean theorem	68	30	3	7
The number π	64	31	4	7
Area of a parallelogram	54	34	13	8
Integers	64	37	0	5
Subtraction of integers	53	42	5	6
Solving linear equations	87	13	0	5
MEAN	57.1	36.6	6.8	

On the average, 57% of teachers emphasized at most one approach. Slightly more than a third emphasized two or three approaches while a small number reported emphasizing 4 or more approaches to a topic. The exceptions to the general pattern were for interpretations of fractions and decimals both of which are review topics. In these two cases, teachers were far more likely to have emphasized two or three approaches.

If the categories of emphasis and use are combined, very few teachers limit themselves to a single approach or strategy. Overall, the split is almost even between using two or three approaches and using four or more. Within the categories of interpretations of fractions and decimals, 91% of teachers reported that they emphasized or used 4 or more strategies. In fact, 55% reported that they emphasized or used at least 8 interpretations of fractions.

The results may be summarized by saying that most teachers emphasize a single concept interpretation or teaching strategy, although a fairly large number emphasize two or three approaches. For some content categories almost all teachers report using a fairly large number of different approaches.

3.8 Influence of Textbooks

In Section 3.6 the reported influence of textbooks on teachers' decisions to emphasize or omit certain instructional approaches was discussed. Besides emphasis and non-use, a third category of partial use was also included for each item. For each item the teacher was asked whether or not the interpretation or strategy was contained in the textbook used with the class. This informatioon was used to define a variable, the influence of textbooks, to measure relationship between emphasis or use of an instructional approach and its presence in the textbooks used.

To compute the value of this variable for each teacher and each content category, the number of interpretations or strategies reported as emphasized or used and contained in the textbook was divided by the number of interpretations or strategies emphasized or used whether or not they were contained in the textbook. This quantity was then multiplied by 10 creating a scale with a range between 0 and 10. A score of zero would indicate that none of the interpretations or strategies used were in the textbook while ten would indicate that they all were present.

Scores of 0-3 indicate low levels of textbook influence on instruction, 4-6 medium levels, and 7-10 high levels. The largest percent of teachers scoring in the high level, 73%, occurred for interpretation of integers. The largest percent of teachers scoring in the low category, 41%, occurred on the item dealing with methods of teaching the Pythagorean theorem. For all teachers and categories, about half the scores were within the high level of influence; in fact, in 26% of the cases a score of 10 was attained.

Thus, while half of the teachers reported a strong relationship between how they taught specific subtopics and the approach used in the textbook, over a quarter reported seldom following the treatment in the book. Since a large number of teachers reported primary reliance on worksheets, the possibility exists that many teachers showing a low level of textbook influence are simply not utilizing a textbook in their classrooms.

One rather firm conclusion relevant to curriculum revision can be made, however. Specific content interpretations

and strategies desired at the level of the intended curriculum will not carry over to the implemented curriculum of many teachers if they are not evident in the textbook or other material provided for use by students. This finding is consistent with the results described in the section dealing with decision-making by teachers.

CHAPTER 4

THE TEACHING OF MATHEMATICS: GRADE 12

The Second International Mathematics Study, as conducted in B. C., included an investigation of the classroom practices used in the teaching of mathematics at the Grade 12 level. The goal of this phase of the project was to determine which topics teachers were actually teaching, the amount of time they spent on these content areas, the methods they employed, and their opinions regarding issues in the teaching of mathematics.

The instruments used in this part of the study included a General Classroom Practices questionnaire and three Classroom Process questionnaires[1]. The General Classroom Processes questionnaire was designed to examine teachers' goals for mathematics instruction, their grouping and evaluation practices, and their opinions regarding teaching practices. The topic-specific questionnaires, on the other hand, were intended to solicit teachers' opinions concerning particular areas of content. The topics dealt with in these questionnaires were:

- Trigonometry,
- Analytic Geometry,
- Algebra,
- Calculus.

The Calculus questionnaire was not used in British Columbia because calculus is not part of the core curriculum for Algebra 12 as outlined in the Curriculum Guide[2]. In fact, very few Algebra 12 teachers teach any calculus topics as part of their courses, even those following the enriched format.

---

[1]A sample of one of the Classroom Process questionnaires for Population B is contained in Appendix E.

[2]A listing of the prescribed content of the Algebra 12 course is contained in Appendix H

The three topic-specific questionnaires were provided to teachers in sealed envelopes. In order to minimize the possibility of the content of the questionnaires influencing their teaching, teachers were asked to complete each one as soon as possible after teaching the topic concerned. Teachers were also instructed to limit their responses on the questionnaires to the one Algebra 12 class which was participating in the international study.

The following aspects of instructional practice were examined in the Population B topic-specific questionnaires:

- specific subtopics taught;
- level of justification given by teachers and expected of students for specific formulas;
- interpretation of specific concepts such as the trigonometric functions;
- instructional methods and organization of content;
- reasons teachers perceived as being influential in their choice of subtopics and in their approaches to concepts and procedures;
- total time allocated to each of the three major topics, to the specific subtopics, and to types of instructional activities;
- use of computational and instructional aids;
- methods of evaluation;
- teachers' opinions regarding content issues.

In addition, the Trigonometry and Algebra questionnaires contained items pertaining to possible contexts from which applications might be drawn.

Although there were a number of similarities between the types of items on the Grade 8 and Grade 12 Classroom Process questionnaires, there were differences as well. For example, the Grade 8 instruments stressed alternative interpretations of concepts and specific teaching strategies. Over 30 such topics were included on the questionnaires, and teachers were asked to indicate the extent to which these examples characterized their teaching. Specific alternatives of this type were provided in only four areas on the Population B instruments: trigonometric functions, multiplication of complex numbers, a property of logarithms, and strategies for solving systems of linear equations.

Another difference between the questionnaires at the two levels was in the analysis of decision-making by teachers. At both levels, teachers were asked why they used or did not use various concept interpretations and teaching strategies. Unlike their Mathematics 8 counterparts, however, Algebra 12 teachers were also asked why they chose to include or not to include particular subtopics in their course. The level of justification teachers provided for theorems and procedures, and whether or not they expected their students to be able to replicate or produce their own proofs and justifications were also investigated in more detail at the Grade 12 level than at Grade 8.

On each of the three questionnaires administered to Algebra 12 teachers, the general topic was sub-divided into a number of subtopics. The Analytic Geometry questionnaire dealt with 14 subtopics, the Trigonometry one with 20, and the Algebra one with 31. Teachers were asked to classify their coverage of these 65 subtopics using the following categorizations:

- taught for the first time,
- reviewed and then extended,
- reviewed only,
- assumed as prerequisite knowledge and neither taught nor reviewed.
- not taught and not assumed as prerequisite knowledge

They were also asked to specify the number of class periods spent on each subtopic, and the reasons why they had or had not taught each topic to the target class.

For each major topic, some of the subtopics were taught by virtually all teachers, others were seldom taught, and the remainder received mixed coverage. Table 4-1 shows the number of subtopics in each of these three categories.

Over one-third of the 65 subtopics were seldom taught, and most of these were within the Algebra area. This compares with the fact that only three of the 74 subtopics on the Grade 8 questionnaires were not widely taught at that level, and it also demonstrates that, at the Population B level, the B.C. curriculum differs widely from that of other countries.

Table 4-1
Coverage of Subtopics
(Frequency)

Questionnaire	No. of subtopics	n≥80%	20%<n<80%	n≤20%
Algebra	31	12	2	17
Analytic Geometry	14	4	7	3
Trigonometry	20	12	5	3
Overall	65	28 (43%)	14 (22%)	23 (35%)

4.1 The Teaching of Analytic Geometry

All of the teachers reported that they taught analytic geometry in their Algebra 12 courses, spending an average of 21 class periods on it. The range of time varied widely, with one teacher reporting six periods devoted to the topic, and another reporting 45 periods. Of the 14 subtopics listed on the questionnaire four were taught by most teachers:

- rectangular coordinates and equations of a line and circle,
- equations and graphs of the conics,
- graphing polynomial functions (curve sketching),
- graphing other functions (trigonometric, logarithmic, exponential, etc.).

Almost all teachers included the first two of these, while 88% taught each of the others. A fifth subtopic, translation of axes, was taught by about two-thirds. Only the first of the five subtopics was generally perceived as review material.

At the other extreme, six subtopics on the questionnaire were generally not taught. Those six are listed below, along with the percent of teachers who did teach them.

- the polar coordinate system (28%),
- parametric equations of a line in a plane (21%),
- topics concerning vector operations such as the inner product (20%),
- analytic proofs of geometry theorems (19%),
- rotation of axes (15%),

- equations in polar coordinates (12%).

Since the Curriculum Guide recommends inclusion of polar coordinates and vectors only in the enriched format for the course, it may be that these topics were taught in classes containing especially able students.

Three subtopics received mixed coverage:

- rational functions,
- equations of lines in 3-space,
- equations of planes in 3-space.

These were taught by 52%, 38%, and 45% of the teachers respectively. The first of these subtopics is listed in the Curriculum Guide as falling under the enriched format, while the others come under the optional category of solid analytic geometry. Since these subtopics are part of the curriculum for only a select sub-population of Algebra 12, they appear to have been taught more often than might have been expected.

Conic Sections

Conic sections and quadratic systems form one of nine core topics in the Curriculum Guide for Algebra 12, and the conic sections portion of the Analytic Geometry questionnaire was highly relevant to actual instruction at this level. Teachers reported spending an average of eight periods on equations and graphs of circles, parabolas, hyperbolas and ellipses.

Virtually all teachers covered graphing conics in standard position and also translated to a centre at (h,k). There was also general coverage of the equations of directrices and asymptotes. However, only 25% of the sample discussed the role eccentricity has in determining the shape of the conics. Even fewer teachers appear to have provided in-depth treatment of the general second degree equation:

$$Ax^2 + Bxy + Cy^2 + Dx + Fy + F = 0.$$

For example, only 4% indicated that they used the discriminant test involving $B^2 - 4AC$ to determine the type and location of a conic defined by any particular equation of this form. Similarly, 3% discussed the determination of the angle of rotation of a conic not oriented in the standard position.

Even if the general second degree equation was not given the level of treatment that might be typical in a calculus or analytic geometry course, conic sections themselves were per-

ceived as important by Algebra 12 teachers. In fact, when they were asked to rate the importance of eight analytic geometry content areas, including:

- manipulation of equations of conics,
- manipulation of equations of lines,
- solutions of locus problems,
- development of graphical techniques,
- development of geometrical intuition concerning spatial relationships,

manipulation of equations of conics was by far the most highly rated. Over 85% rated it as being of primary importance to their programs. The other areas were rated as being of primary importance by an average of 20% of the sample.

Solid Analytic Geometry

In B. C. little solid geometry is taught as part of the Algebra 12 course. Another course, Geometry 12, is taken by 6% of the Grade 12 population, and although the Curriculum Guide does not include a detailed outline for this course, some solid geometry is included in it. Just over 50% of the teachers stated that no solid analytic geometry was taught in their Algebra 12 classes, and it does not appear that these teachers treated this area informally either. When they were asked to rate the importance of the development of spatial intuition in their courses, 50% rated it as a goal of primary or secondary importance, while the others rated it as being of little or no importance.

Those teachers who did teach some three-dimensional geometry reported spending an average of between 2 and 3 periods on it. The coverage included graphing points and planes in 3-space. Sketching cross-sections of solids and sketching spheres and quadric surfaces were seldom included. The equations of lines and planes in space were covered by most teachers. A minority, 13 teachers, taught the formula for calculating the distance from a point to a plane.

Overall, Algebra 12 teachers were evenly divided between those who spent no time on solid geometry and those who generally spent several days on it. While a few teachers provided fairly extensive coverage, it is reasonable to assume that most Algebra 12 students acquire little, if any, expertise in solid geometry. This is quite in keeping with the place given to solid geometry in the Curriculum Guide.

Vectors

The data show that, except for certain trigonometric applications, the topic of vectors is not a significant one in most Algebra 12 classrooms. First of all, only 20% of teachers indicated any inclusion of topics concerning vector operations such as addition and the inner product in their courses. When asked to specify the number of periods spent on the topic, 78% responded with zero or left the item blank. An additional 20% reported spending between one and five periods on vector operations. When asked the broader question, "What percent of the study of analytic geometry was from a vector viewpoint?", just under 70% indicated none or left the item blank. Slightly over 20% indicated that between 1% and 10% of their instruction was vector-oriented. Of the remaining 10 teachers, three reported that half or more of the time they spent on analytic geometry was vector-based.

Teachers were also asked how they developed or reviewed the formula for the equation of a line. Just 3% stated that it followed a preliminary development of the ideas of vector algebra. An additional 7% first developed the linear equation algebraically and then considered a vector approach. Almost 90%, however, agreed with the statement that they developed all topics in analytic geometry strictly from an algebraic viewpoint without any use of vectors. Similarly, on the Trigonometry questionnaire, only 3% of teachers indicated that they developed the Law of Cosines using vector methods. Vectors were seldom taught in the sense of a vector space development with an emphasis on formal properties and operations such as the cross- or inner-product. Likewise, vectors were not used as a vehicle for explaining basic results in analytic geometry. This lack of emphasis on vectors in Algebra 12 is consistent with vectors in the plane being included as a topic in the enriched format of Algebra 12 only, and vectors in 3-space being categorized as an optional enrichment topic.

While vectors as a separate topic is suggested for able students only, it is listed as a core subtopic in the Curriculum Guide under the heading of trigonometry. An examination of the references to textbooks in this regard shows that they follow a traditional treatment emphasizing force and velocity applications. This explains a seeming contradiction in the questionnaire results. While few teachers reported any general coverage of vectors as outlined above, almost 90% stated that they used vector applications in trigonometry. This is another example of how closely Algebra 12 teachers follow the recommendations in the Curriculum Guide.

4.2 The Teaching of Trigonometry

Every Algebra 12 teacher in the survey reported teaching topics in trigonometry, spending an average of 29 periods on the subject. The range in the amount of time allocated to trigonometry was large, from 10 to 56 periods. Slightly over one fifth of the sample felt they spent the equivalent of a full semester on trigonometry with the rest indicating less coverage. In fact, two-fifths favored a return to a semester-long trigonometry course, while about 30% were undecided. Nearly 90% disagreed with the statement that trigonometry is currently overemphasized in the curriculum.

Of the 20 subtopics listed on the Trigonometry questionnaire, 12 were taught by over 80% of teachers. In fact, the first 10 of the 12 subtopics listed below were taught by 95% or more:

- definition of the 6 trigonometric ratios in right triangles;
- solution of right triangles given sides or angles;
- standard position for angles with vertex at the origin;
- definition of the trigonometric functions using a unit circle;
- periodic properties of the trigonometric functions;
- graphs of the sine, cosine and tangent functions;
- applications of the Law of Sines, Cosines, or Tangents;
- sum and difference of angles identities;
- double and half angle identities;
- solution of trigonometric equations;
- graphs of the secant, cosecant, and cotangent functions;
- graphs of general trigonometric functions such as $y = A\sin(Bx+C)$.

Of the remaining eight subtopics, only one was taught in a majority of classes: inverses of the sine, cosine and tangent functions by 65% of the teachers. The subtopics which are listed below were taught by less than half of teachers. The

number in parentheses indicates the percent of teachers who did teach that subtopic.

- inverses of the secant, cosecant, and cotangent functions (37%);
- graphs of the Arcsine, Arccosine and Arctangent functions (35%);
- polar coordinates (31%);
- definition of the trigonometric functions using a wrapping function with the unit circle (26%);
- graphs of the Arcsecant, Arccosecant and Arccotangent functions (19%);
- graphing in polar coordinates (17%);
- De Moivre's theorem (17%);

While almost two-thirds of the teachers did discuss the inverses of the sine, cosine, and tangent functions, only about half that number graphed these functions or discussed the other three inverses at all. This is consistent with the fact that inverse trigonometric functions are part of the enriched course, as is De Moivre's theorem.

Although trigonometry is one of the nine core units in the Algebra 12 curriculum, it is also one of the eight core units in the preceeding course, Algebra 11. Thus, Algebra 12 students should begin the year with some knowledge of this area. An inspection of the Algebra 11 curriculum and the treatments given to trigonometry in the two approved textbooks reveals that students should be exposed to both the right triangle and the unit circle approaches to the trigonometric functions in Algebra 11. In fact, while 95% of the Algebra 12 teachers surveyed felt that their students began the year with at least a minimal understanding of the ratio approach, only 33% were equally optimistic regarding the unit circle approach. Similarly, while solving right triangles and graphing the sine, cosine and tangent functions are both listed as core topics for Algebra 11 and are covered in the prescribed textbooks, 42% and 76% of the Algebra 12 teachers respectively stated that they were teaching them as new, not review, material in their courses. This may indicate that Algebra 12 teachers feel that students have difficulty with trigonometry in Algebra 11, or that less coverage than is suggested in the Curriculum Guide at that level is, in fact, done in Algebra 11.

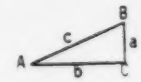
Interpretations of the Trigonometric Functions

Three interpretations of the sine, cosine, and tangent functions were described on the Trigonometry questionnaire:

- ratios in a right triangle,
- coordinates and ratios using a unit reference circle,
- a variant of the unit circle interpretation using a wrapping function.

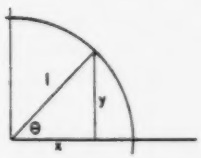
These interpretations are shown in Figure 4-1.

Ratio Interpretation: The three basic trigonometric functions are defined and illustrated as ratios of the lengths of sides of a right triangle.



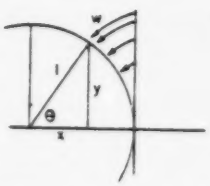
sin A = a/c
cos A = b/c
tan A = a/b

Unit Reference Circle: The three functions are defined and illustrated as in the ratio interpretation except the triangle has hypotenuse 1 and point B is located on a unit radius circle with centre at the origin of the coordinate plane. Here the functions map the real angle measures into real numbers.



sinθ = y, cosθ = x, tanθ = y/x

Wrapping Function: First a wrapping function W is defined that maps the real number line onto the circumference of the unit circle. The trigonometric functions are then defined so that they map the real number angle measures (modulo 2π) to the real numbers.



sinθ = y, cosθ = x, tanθ = y/x

Figure 4-1. Interpretations of the three basic trigonometric functions.

It has aleady been noted that teachers were more likely to feel that their students would have some prior understand-

ing of the ratio interpretation than of the unit circle interpretation. However, in their instruction both interpretations were apparently emphasized equally. Virtually every teacher employed both interpretations to some extent. Almost three-quarters emphasized the ratio approach, while nearly four-fifths emphasized the unit circle approach. Futhermore, they allocated virtually the same number of periods to each approach.

It might have been expected that Algebra 12 teachers would have stressed the unit circle approach more than the right triangle approach. The Curriculum Guide explicitly suggests that stress be placed on the unit circle and the term "circle function". Also, according to teachers, students enter Algebra 12 knowing less about the unit circle approach than the right triangle approach. The data, however, indicate that, if the unit circle approach is stressed more than the right triangle approach, it is by a very narrow margin.

The third interpretation of the trigonometric functions, using the idea of a wrapping function, is very similar to the basic unit circle approach and is one that became somewhat popular in the New Math era. Only 25% of teachers used this interpretation at all, with 10% indicating that they placed emphasis on the wrapping function. Of those teachers who did employ the wrapping function, most felt that their average students would not be able to understand the approach thoroughly. In contrast, most teachers felt that average students could understand both the unit circle and the ratio interpretations. It might seem surprising at first that so few teachers included wrapping functions and that expectations of average student understanding were so low, given the inclusion of this interpretation in a number of textbooks. Inspection of the Curriculum Guide and the prescribed textbooks, however, clarifies the situation. While the term wrapping function is not used in the Guide, one of the suggested textbooks for the enriched format of the course does employ this approach. On the other hand, neither textbook suggested for the core format uses wrapping functions.

Curve-Sketching in Trigonometry

The graphing of trigonomtric functions is generally part of the Algebra 12 course, but it is not a section that receives strong emphasis. This contention is supported in several ways by the data from the questionnaire. First of all, teachers were asked to rate the importance of six aspects of trigonometry to their courses:

- skill in solving for missing parts of right triangles,

- skill in proving trigonometric identities,
- skill in solving equations involving trigonometric functions,
- developing trigonometric function curve-sketching skills,
- developing algebraic techniques for dealing with trigonometry problems,
- applying trigonometric skills to solve application problems.

Between 70% and 80% of teachers rated each of these aspects of trigonometry as being of primary importance to their teaching, except for curve-sketching. In fact, less than one-third of the teachers considered curve-sketching as an area of primary importance. The algebraic and computational aspects of the subject were emphasized, as was proving identities, but the visual aspect was not.

Data on time allocation reinforces the contention that graphing in trigonometry was not usually emphasized. For example, while teachers spent an average of eight periods on proofs of identities and seven on applications, just four periods were spent on graphing. In fact, 30% allocated two periods or less to curve-sketching. Only a cursory treatment of sketching the basic functions, the inverse functions, and general functions can be provided in such a limited time. Although 96% of teachers agreed with the statement that students should be expected to graph both the basic and general trigonometric functions, between 20% and 30% did not teach graphing of general functions such as

$$y = A\sin(Bx+C).$$

In graphing general functions, the concepts of periodicity and amplitude must be understood, but only a fifth of the Algebra 12 teachers agreed with the statement that a significant portion of the study of trigonometry should be devoted to these ideas.

While most teachers did report spending one or two periods on graphing the secant, cosecant, and cotangent functions, only 40% agreed that their students should be expected to graph the inverse trigonometric functions and to relate their graphs to the graphs of the basic trigonometric functions. Thus, apart from graphing the six basic trigonometric functions, the data strongly suggest that trigonometric curve-sketching is a low priority topic in British Columbia.

4.3 The Teaching of Algebra

Algebra 12 teachers spend almost as much time teaching algebra as they spend on analytic geometry and trigonometry combined: they reported spending an average of 49 class periods on this topic. The reported variations in time allocation was startling, with as few as seven periods and as many as 96 periods being devoted to it. The standard deviation was 28 periods.

The scope of the Algebra questionnaire was extremely wide, and for that reason ten broad subtopics or content strands were first specified, and were divided into a total of 31 narrower subtopics. It turned out that half of these content strands were almost never relevant to actual instruction in British Columbia. Similary, several aspects of algebra that are included in the B. C. Curriculum Guide, notably the core unit of sequences and series, were not mentioned in the Algebra questionnaire.

The fact that this questionnaire was only partially relevant to the B. C. curriculum accounts for an interesting anomaly. While teachers reported spending an average of 49 periods on what they considered to be algebra, the total time reportedly spent on all 31 subtopics was only 26 periods. This means that almost half of the algebra covered in British Columbia is in areas not included on the questionnaire. In the case of the other two Population B questionnaires, Analytic Geometry and Trigonometry, and the five Mathematics 8 questionnaires, the overall time reported for each major topic and the subtopic totals were nearly equal. In that sense, then, the Algebra questionnaire was easily the one least suited to the B. C. curriculum, except for the Calculus questionnaire which was not used at all.

Of the 10 content strands on the Algebra questionnaire, four were taught in at least 95% of Algebra 12 classes:

- polynomial functions,
- logarithmic functions,
- exponential functions,
- complex numbers.

These four content strands were divided into the 15 subtopics listed below. The numbers in parentheses represent the percent of teachers who included that subtopic in their course.

- sythetic division, rational roots (100%)

- laws of logarithms (100%);
- laws of exponents (100%);
- complex roots of quadratic equations (100%);
- applications of logarithms (98%);
- solving exponential equations (98%);
- graphing complex numbers on rectangular coordinates (94%);
- graphing exponential functions (88%);
- graphing logarithmic functions (88%);
- applications of exponential functions (87%);
- graphing inequalities (74%);
- solving quadratic inequalities (74%);
- natural logarithms (44%);
- polar coordinate representation for complex numbers (20%);
- De Moivre's theorem and roots of unity (16%).

It is not surprising that polar coordinates and De Moivre's theorem were generally not widely taught since these are listed as enrichment topics in the Curriculum Guide. The topic of natural logarithms is also listed as an enrichment topic, but almost half the teachers apparently felt that all Algebra 12 students should study it. Those teachers who omitted the graphing of inequalities and solving quadratic inequalities usually assumed that their students were already competent in those areas.

Just as four of the ten content strands for algebra listed on the questionnaire were highly relevant to the Algebra 12 curriculum, five others were highly irrelevant:

- elementary linear algebra--matrices and determinants;
- algebraic structures--groups, rings, and fields;
- counting, permutations, and combinations;

- probability;
- statistics.

Linear algebra, an optional topic in the Curriculum Guide, was taught by 12% of the teachers. The other four areas were almost never taught. No more than 4% of the teachers included any of them in their courses.

The remaining content strand was logic, sets, and operations on sets, and about one-fifth of the teachers reported some coverage of this area. Most of those teachers who did not explicitly review sets and set notation assumed that their students were already familiar with these topics. They did not, however, assume that their students would understand elementary logic and inference. In spite of this, only 11% included it in their courses.

Exponential and Logarithmic Functions

The study of exponential and logarithmic functions and algebraic manipulations with exponents and logarithms were considered to be of significant importance by almost all teachers. On the average, 14 periods were allocated to these topics, not including time for review or tests.

Before the curricular reforms of the New Math era and the advent of calculators and computers, logarithms were studied primarily for computational applications. Logarithms were also frequently used to explain the basis for computation with a slide rule. Curricular reform materials emphasized the importance of logarithmic functions and their properties. It is quite surprising, therefore, to find that 13% of Algebra 12 teachers reported emphasizing a computational approach to the study of logarithms as contrasted with a functional approach. Moreover, another 36% indicated that in their teaching, they struck a fairly even balance between those two options. The fact that almost every teacher also reported that students used calculators in their class makes it difficult to understand why any emphasis was placed on logarithmic computation. The fact that this use of logarithms has been retained in the Algebra 12 core curriculum undoubtedly explains this practice in part.

One of the basic properties of logarithms is the fact that

$$\log ab = \log a + \log b$$

for all positive values for a and b. Teachers were asked to indicate the degree to which they employed the approaches

listed in Figure 4-2 in teaching this property by selecting one of the following categories:

- emphasized;
- used, but not emphasized;
- not used.

Exponent Base: Logarithms are defined as exponents. Students abstract the generalization from observing, and working with, patterns such as:

$$4 \times 32 = 2^2 \times 2^5$$

$$= 2^7 = 128$$

Here log ab = log a + log b is considered a restatement of:

$$10^a \times 10^b = 10^{a+b}$$

Inverse Function Base: A logarithmic function is defined as the inverse of the exponential function $f(x) = 10^x$. Consider the graph of the log function. It is observed for several specific problems that the ordinate at ab is equal to the sum of the ordinates at x = a and at x = b. Thus ab = log a + log b.

Area Under a Curve Base: Logarithmic functions are defined in terms of area under curves of the form $f(x) = k/x$. ($y = \log x$ is associated with $k = 0.434$) Log b is then defined as the area under the graph of $f(x)$ for $1<x<b$. By counting squares on a fine grid paper for several specific problems, students form the generalization that the area under the curve from 1 to ab is the sum of the areas under the curve from 1 to a and from 1 to b.

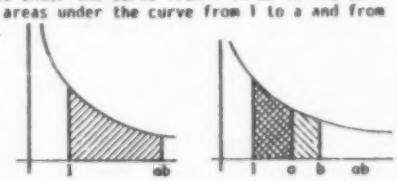


Figure 4-2. Teaching log ab = log a + log b.

Little variation was found in the way teachers approached this property. Every teacher employed the exponent-base approach with over 80% indicating that they had emphasized this method. A third of the sample did employ the inverse function interpretation, but primarily at the level of use rather than emphasis.

The last approach to this property, although it does provide an intuitive and visual justification for the property, requires a new interpretation of the concept of logarithm--one that is not closely related to the familiar concept of exponent. While logarithms interpreted as areas may be common in calculus courses, this approach to the property held little appeal for Algebra 12 teachers. Only 4% made any use of it.

The treatment given to logarithms was examined in another way. Teachers were asked to estimate the percent of the time devoted to logarithms that involved graphs relating the visual aspects of the logarithmic functions to the symbolic-algebraic representations. Over 80% estimated this figure at 10% or less. While it might be argued that these items on the questionnaire items suffered from somewhat obscure wording, this result is consistent with the relatively light emphasis given to graphing in trigonometry.

Complex Numbers

Almost all Algebra 12 teachers reported teaching material on complex numbers, complex roots of quadratic equations, and rectangular coordinate representation of complex numbers. At least 80% covered operations with complex numbers through multiplication, while 47% reported specific inclusion of the division theorem:

$$\frac{a + bi}{c + ji} = \frac{ac + bd}{c^2 + j^2} + \frac{(bc - aj)}{c^2 + j^2}i$$

About 20% of teachers reported teaching polar-coordinate representation of complex numbers, De Moivre's theorem, and n-th roots of unity.

Methods of teaching multiplication of complex numbers were investigated in some detail. Teachers were provided with the four interpretations shown in Figure 4-3 and asked if they emphasized, used, or did not use them.

The first approach to multiplication, the one emphasizing the basic definition, $i^2 = -1$, was clearly preferred: 64% emphasized it and another 19% used it. By contrast, the second interpretation, utilizing trigonometric functions, was employed in only 10% of classes. While this is the only method that can be used to give geometric meaning to the operation, it does require the use of polar coordinates.

The third and fourth approaches are similar, since both begin with a defining formula for complex number multiplication and then proceed to justifing the algorithm on the basis of the field properties. The first of these was employed by a third of the teachers, usually at the level of use rather than emphasis. The last method, one which omits i from the notation and is thus quite abstract, was used by only 10%.

From $x^2 + 1 = 0$, we define $i^2 = -1$ and then use the distributive property to give a rationale for the product:

$(a + bi)(c + di) = ac + bdi^2 + bci + adi = (ac - bd) + (bc + ad)i$

Multiplication is defined by stating $(a+bi)(c+di) = (ac - bd) + (bc + ad)i$ and then it is verified that multiplication in $\mathbb{C}$ satisfies the various algebraic properties of a field.

Multiplication is considered as a rotation transformation followed by a dilation (stretch or shrink) transformation.

If $z_1 = a + bi = r_1 (\cos \alpha + i \sin \alpha)$

and $z_2 = c + di = r_2 (\cos \beta + i \sin \beta)$,

then $z_1 \cdot z_2 = r_1 r_2 (\cos(\alpha + \beta) + i \sin (\alpha + \beta))$

dilation rotation

Multiplication is defined as follows: If $z_1 = (a,b)$ and $z_2 = (c,d)$, then $z_1 \cdot z_2 = (ac-bd,bc+ad)$. After the definition is stated, the operation is checked to see if multiplication thus defined satisfies the algebraic properties of a field.

Figure 4-3. Interpretations of multiplication of complex numbers.

Probability and Statistics

As was mentioned earlier, almost none of the Algebra 12 teachers reported teaching combinatorics, probability, or statistics in their courses. Results on more specific questions confirmed this general assessment. For example, only one of the 95 teachers in the sample reported teaching any of the following topics:

- permutations,
- empirical probabilities,
- finding probabilities of (A or B) or of (A and B),
- conditional probabilities.

Not only were these probability subtopics not taught, they were not assumed by any teachers as prerequisite knowledge for their courses. Other subtopics were taught only slightly more often. Each of the following was taught by fewer than 5% of the teachers:

- the Fundamental Counting Theorem,
- combinations,
- variance and standard deviation,

- measures of central tendency.

Combinatorics, probability, and statistics at this grade level are apparently seen as the domain of the Probability and Statistics 12 course and not of Algebra 12. This view is consistent with the treatment of this content area in the Algebra 12 course outline: it is listed last among the four optional topics.

Probability and statistics are also optional topics in a crowded curriculum at the Mathematics 9 and 10 levels, so it is highly unlikely that much attention is given to them at these earlier grade levels. While an entire course is devoted to probability and statistics at the senior secondary level, that course is currently taken by only 3% of the student population. Algebra 12, by contrast, is taken by almost 40%. The restriction of even the most basic concepts of probability and statistics to such a small proportion of the student population would appear to be a weakness of the present curriculum.

4.4 Formulas: Their Justification and Use

The inclusion or exclusion of formulas by teachers in their Algebra 12 courses is remarkably uniform across British Columbia. This assertion is supported by the responses of all 95 teachers with regard to 32 formulas included in the three topic-specific questionnaires.

The results showed a clear dichotomy of usage. Of the 32 formulas, 11 were taught by 88% of the teachers on average, while the remaining 21 were taught by an average of only 15% of the teachers. The ones taught by most teachers are listed below:

- $\cos(A - B) = \cos A\cos B + \sin A\sin B$
- $\cos 2A = 2\cos^2 A - 1$
- $\sin 2A = 2\sin A\cos A$
- $\tan 2A = 2\tan A/(1 - 2\tan^2 A)$
- $\cos(A/2) = \pm(1 + \cos A)/2$
- $\tan(A + B) = (\tan A + \tan B)/(1 - \tan A\tan B)$
- formula for the distance between two points in the plane.
- formula for the equation of a hyperbola from the focus-directrix definition of a hyperbola.

- log x = y and log x = z if and only if b = a
- for positive integers x, p, and q, x = (x) = x
- for all real numbers, $a^2 = |a|$

The 21 formulas which were seldom taught included some of the more advanced ones from analytic geometry and trigonometry. Among them were the following:

- sinA + sinB = 2 sin[(A + B)/2]cos[(A - B)/2]
- formula for the angle between two lines

While the treatment accorded particular formulas is of interest and could be used to expand the earlier discussions in this chapter, the primary reason for considering these particular formulas was not to explore whether or not teachers taught them. It was rather to investigate the level of justification teachers provided for formulas in general and their expectations of students in this regard. Teachers were asked to categorize their presentation of each formula using the categories below:

- I gave a formal proof for the formula,
- I stated the formula and gave and informal justification (graphic or verbal),
- I stated the formula and used it without any justification,
- I did not cover the formula or discuss it with my class.

Teachers were also asked to indicate what they expected their students to be able to do with formulas. For each formula they taught, they selected one of the following choices to describe their expectations of students:

- prove and apply the formula,
- informally justify and apply,
- recall and apply formula,
- apply formula when given.

While teachers' responses did differ to a certain extent among formulas, the variations were not large. For the 11 frequently taught formulas, 60% of teachers who taught them also

proved them for students. If teachers did not prove a particular formula, they reported providing at least some justification. For the 11 frequently taught formulas, teachers indicated informal justification 32% of the time. These formulas were, then, either proved formally or justified informally 92% of the time. Justification was provided somewhat less often for the 21 other formulas in those few classes where they were taught.

Although teachers justified the reasonableness of formulas with some level of proof or demonstration, they expected student proof or justificaton far less often. For the commonly taught formulas, student proofs were expected by an average of 8% of teachers. Less formal justification was expected by 19%. While formal proofs of formulas can be memorized and replicated as unthinkingly as the formulas themselves, it seems unusual that teachers apparently spent so much time justifying formulas in class and yet expected so little proof or justification from students.

The usual teacher expectation of students was an ability to recall formulas from memory and to apply them in problem-solving situations. Students were expected to show such competence by 52% of teachers for the commonly taught formulas. As stated above, proof or informal justification was expected by 27%. In the remaining 21% of cases, neither justification nor recall ability was required of students. Instead, they were expected to be able to use the formulas when given. The data suggest that the ability to apply, but not necessarily recall, formulas was expected somewhat more often for trigonometric formulas than for formulas from other areas.

4.5 The Use of Instructional Time and Other Resources

Wide variations in the amount of time teachers reported spending on analytic geometry, trigonometry, and algebra have already been noted in this chapter, and that information is summarized in Table 4-2. The table contains the number of class periods allocated by various percentiles of Algebra 12 teachers. For example, 25% of teachers reported spending 15 periods or less on analytic geometry while 10% spent more than 32 periods on that topic. The reported variations are especially large for algebra.

Many of the Algebra 12 classes in the sample were semestered, and it is reasonable to expect that such classes would have less time for mathematics than would year-long classes. A more refined analysis of the data is clearly required before any firm statements about the extent of variation in time allocated by teachers for the three broad topics can be made. Initial inspection of the data suggests, however, that substantial variations do occur. Such variations might also have

Table 4-2
Time Allocated to Topics

	Percentiles						
	10th	25th	50th	75th	90th	Mean	S.D.
Analytic Geometry	12	15	20	25	32	21.3	8.3
Trigonometry	17	23	30	35	40	29.4	9.5
Algebra	20	28	40	80	93	48.6	27.7

implications for achievement by students.

Information was also obtained regarding the use of instructional aids for Algebra 12, as well as the utilization of various potential sources of applications. Table 4-3 shows the percent of teachers who reported use of various aids.

Table 4-3
Use of Instructional Aids

	Percent
Hand Calculators	97
Microcomputers	16
Film Loops	1
Movies	1
Manipulative Models	22
Wire, Plastic or Plaster Models	43
Student Constructed Paper Models	10
Prepared Overhead Transperancies	38

Calculators were used in almost every Algebra 12 class, and almost a third employed programmable calculators. Microcomputers were used in some way in 16% of classes. This rate of use is likely to increase significantly in the future given the current rate of development of educational software. About 40% of teachers reported that they used prepared overhead transparencies and models. Almost no one found the films and film loops which were available to them to be valuable instructional aids.

The final type of resource usage explored in this study was sources of applications, and these results are shown in Table 4-4. Of the print resources, nearly every teacher used the basic textbook as a source of word problems, with most

also using supplementary textbooks. Journal articles were useful to about 20% of teachers, and a few indicated the Curriculum Guide as source, although that document contains no such problems. Slightly over 50% of teachers indicated that they created some of their own application problems while a small number, 13%, found professional meetings useful in this regard.

Table 4-4
Sources of Applications

Source	Percent
Problems from the textbook	98
Supplementary textbooks	78
Problems created by the teacher	56
Articles in professional journals	21
Problems obtained at a provincial or NCTM meeting	13
The Curriculum Guide	6
No application problems were used	1

4.6 Influences on Content Decisions

An area of special interest on the questionnaires was the potency of various factors in teachers' decision-making regarding course content and teaching methods. The origins of such influences were probed in each of the three topic-specific questionnaires. First, teachers were asked why they had taught the subtopics which they did and why they had excluded others. For each subtopic, teachers were provided with two lists of eight factors which might have influenced their decisions, one list to be used if the subtopic had been included, another if it had been excluded. Two "primary reasons" were to be given for each subtopic decision. For topics they had included teachers did, in fact, list two reasons 87% of the time. For topics they had not taught this figure dropped to 56%.

The second area in which teachers' reasons for instructional decisions was explored was in the domain of the interpretation of mathematical concepts. Given, for instance, that multiplication of complex numbers was taught, how did teachers proceed? Did they rely on the definition that $i^2 = -1$ and an algebraic argument, or did they introduce polar coordinates for geometric meaning and justification? For this example four alternatives were provided and teachers were asked if the approach was:

- emphasized as a primary explanation,
- used but not emphasized,
- not emphasized.

Table 4-5 lists the response options and results for positive decisions: why teachers chose to include particular subtopics and why they chose the interpretations they did. The table shows the relative strength that each of the eight factors had across all subtopics and all concept interpretations. For example, the fact that a given subtopic was listed in a curriculum guide was indicated by teachers 79% of the time as their first primary reason for including that subtopic in their course. Curriculum guides were apparently highly influential documents in teachers' decisions regarding which content to teach. On the other hand, teachers almost never indicated that they selected subtopics because their students would find them easy to understand. They did report, however, that they frequently selected interpretations of concepts on the basis of how easily they would be for students to understand. This motive accounted for 29% of all first primary reasons given for such decisions and 18% of all second primary reasons.

Table 4-5
Reasons for Teachers' Choices
(Percent)

	Subtopics		Interpretations	
	1st	2nd	1st	2nd
Well-known to me	3	3	9	7
In a curriculum guide	79	7	33	15
Easy for students to understand	2	4	29	18
Enjoyed by students	1	4	0	3
Related to math of prior grades	4	6	10	17
Useful in math of later grades	9	54	13	20
I was taught it was appropriate	0	2	1	2
Emphasized in student text	2	18	4	16

Examining each of the eight factors for its influence on teachers' decisions to include subtopics in their courses, it is apparent that only three of these potential factors were most salient:

- in B.C. or local curriculum guide,
- useful in math of later grades,
- emphasized in student text.

In fact, the first factor is highly influential, the second considerably less so, and the third of less importance still. It may seem surprising that curriculum guides were rated a much stronger influence than were textbooks. While this was what teachers reported, it should not be inferred that they felt that guides were more necessary than textbooks. On the General Classroom Practices questionnaire teachers were asked to indicate how difficult it would be to do without various sources of information, and over 80% indicated that it would be difficult to teach without a textbook, while only a third rated the official syllabus as equally essential.

An examination of Table 4-5 shows that the importance of factors on choices of concept interpretations were not the same as their influence on choice of subtopic. Six factors showed some significant degree of impact on teachers' decisions regarding which interpretations of particular concepts to use. The most important of these were:

- in B.C. or local curriculum guide,
- easy for students to understand ,
- useful in math of later grades,
- related to math of prior grades,
- emphasized in student text,
- well known to me.

Teachers' perceptions of the ease with which students would understand a concept was more powerful in helping teachers to select interpretations of concepts than to decide whether or not to include the topic at all. In general, the Curriculum Guide offers little guidance at the level of concept interpretation, but perhaps the teachers reporting this factor as influential were considering the references to textbook pages contained in it and hence to the interpretations in textbooks.

Teachers' decisions were also influenced, although to a lesser degree, by the relation that particular teaching approaches had to mathematics learned earlier or mathematics to be learned in later grades. The textbook itself and teachers'

familiarity with specific approaches were minor influences in these decisions.

Teachers were also asked why they made negative decisions; that is, why they excluded specific subtopics and why they did not use specific interpretations of concepts in their teaching. The eight response categories provided were analogues of the categories used for positive decisions. For example, the factor "well known to me" which could be selected for positive decisions became "not well known to me" for negative decisions. Table 4-6 summarizes teachers' perceptions of why they excluded subtopics or avoided certain approaches. For subtopic exclusion only two factors showed strong influence:

- not in B.C. or local curriculum guide,
- not emphasized in student text.

Table 4-6
Reasons for Negative Decisions
(Percent)

	Subtopics		Interpretations	
	1st	2nd	1st	2nd
Not well-known to me	1	4	18	11
Not in a curriculum guide	90	6	51	10
Hard for students to understand	2	13	17	14
Disliked by students	0	2	0	6
Not related to math of earlier grades	1	11	5	8
Not useful in math of later grades	1	4	1	1
I was taught it was not appropriate	0	5	0	1
Not emphasized in student text	4	55	8	48

Both of these factors were stronger for negative decisions than their counterparts were for positive decisions. In fact, in almost every case if a teacher did not teach a particular subtopic the first reason given was because it was not in a curriculum guide. Algebra 12 teachers apparently are very reluctant to teach subtopics not contained in the official Curriculum Guide. In over half the cases, teachers said that their second reason for not teaching a subtopic was due to the lack of emphasis given it in textbooks. This may well reflect a view that students need a written reference for content covered in class.

Four factors predominated in teachers' decisions to not use specific interpretations of concepts:

- not in B.C. or local curriculum guide,
- not emphasized in student textbook,
- hard for students to understand,
- not well known to me.

As with subtopics not taught, teachers avoided concept interpretations if they were not in the syllabus or textbook. They were also somewhat influenced by perceptions that certain interpretations would be difficult for students. The option "not well known to me" also showed some strength. It should not, however, be inferred that teachers necessarily tended to avoid using strategies just because they were not thoroughly familiar with them. Rather, the relative strength of this factor may well mean that some of the concept interpretations that appeared on the questionnaire were new to the participating teachers.

Two of the eight potential factors included in the topic-specific questionnaires were almost never selected as reasons for content decisions. The enjoyment or dislike that students might feel for a subtopic or an approach was never a decisive factor in teachers' decisions. Teachers were, by contrast, highly influenced by the ease or difficulty an approach might entail for student understanding. "I was taught it was appropriate" and its negation were also rarely selected by teachers.

Overall teachers reported that curriculum guides were strong factors in subtopic selection and in the choice of teaching strategies. They also included subtopics in their courses because such content would be useful in future mathematics courses and excluded others which they felt were not adequately treated in student textbooks. These same factors were influential in the selection of ways of interpreting particular concepts. Other factors were also important, particularly teachers' perceptions of how difficult a given approach would be for students.

4.7 Organization of Instruction

Most of the items on the topic-specific questionnaires dealt with approaches to and opinions regarding very specific mathematics content. A few items on each instrument, however, were directed towards the more general aspects of classroom organization. In particular, methods of instruction, forms of differentiation between students in homework assignments, and

methods of evaluation were investigated. The teaching patterns reported differed only slightly among topics, and, for that reason, all three are discussed together.

As might be expected, lecturing was a common form of instruction, accounting for an average of 40% of all class time. Few teachers reported lecturing less than 25% or more than 60% of the time in Algebra 12. On the average, a quarter of each class period was spent discussing homework, with teachers reporting that they did about twice as much talking as their students. About 20% of each period was typically allocated for individual student work. The practice of having students work in small groups was not commonly used. Two-fifths of the teachers included no small group work, while the others reported devoting no more than 10% of the class time to this activity.

A majority of Algebra 12 teachers taught the same content to all students in their classes and gave all students the same homework assignments. Almost 70% agreed that they taught all students in their target class the same mathematics at the same pace. Slightly over 20% said that they individualized the pace of insruction but not the content, while the remaining 10% indicated that they varied the course content across students or groups of students. While only 10% of teachers had different students study different content within their classes, a third did indicate that some students were assigned exercises or problems that they would not expect other students in the class to do. Most teachers, however, gave all students the same assignments.

Teachers were also asked to specify the methods of evaluation they used in grading students' work. Every teacher used chapter tests, and over a fifth indicated that they gave daily quizzes. Less frequent quizzes were more typical. Less than 20% of the sample included the grading of daily paper work in their evaluation of students, and one teacher graded student boardwork. About 10% used student projects in their evaluations.

In general, teacher-talk was the most common activity reported in Algebra 12 classes. This involved lectures and discussion of homework. The remaining class time was typically allocated for students to work individually on homework. While in almost every class all students studied the same topics, a fair number of teachers did differentiate between students in assigning homework. Chapter tests and, to a lesser extent, quizzes formed the usual basis for the evaluation of student progress. Homework was less commonly a factor, and projects were rarely used in the evaluation process.

CHAPTER 5

STUDENTS' ACHIEVEMENT: GRADE 8

All of the achievement items were expressed in multiple-choice format with five options per item. In administering the tests teachers were asked to make it clear to students that some items might not be familiar and that they should not spend time on such items. The use of calculators, rulers, and protractors was not permitted.

The items were contained in two different types of instruments: the "core" test, and a set of four "rotated" forms. In the first six weeks of the 1980-81 school year all students in the sample of classrooms completed the core test. This test consisted of 40 randomly distributed items, with eight items in each of five categories: Algebra; Fractions; Geometry; Measurement; and Ratio, Proportion, and Percent. Near the end of the academic year or semester all students were again administered the core test. As a result, the analysis and reporting of gains in achievement on the five topics are based on returns from 87 classrooms and 2000 students.

In order to obtain satisfactory measures of achievement across the entire Grade 8 Mathematics curriculum, each student also responded to one of four "rotated" forms near the end of the year or semester. Each form contained 35 items representing the five core categories, and two other categories: Arithmetic; and Probability and Statistics. The proportion of items in each category varied across forms. Teachers received the forms in pre-packaged bundles so that the assignment of forms to students was random. In a typical class of 20 students, five would have written each form. A total of 2138 students from 86 classes responded to the rotated forms.

Students were given a maximum of 50 minutes to respond to each instrument. Because the items did not have an "I don't know" response, at first glance it was not clear what interpretation should be made of an item which was not answered. That is, was the answer not known, or did the student not have enough time to consider the question? To determine whether the time allowed was sufficient for most students, several analyses were undertaken. The percentage of students who failed to respond to each item was graphed, and no trend toward increasing non-response near the end of the test was evident. To confirm this, the moving average of the non-response rates to three successive items was determined. The same result was obtained, although the response rates for the final five items on the core pretest looked slightly suspect. However, inspection of the gain

scores on the final five items did not show suspiciously large gains for those items, that is, it did not seem as if pretest scores had been artificially depressed owing to time constraints. For the shorter rotated forms there was no indication that non-response rates increased near the end of the forms. In general, the conclusion can be drawn with considerable confidence that a student who did not respond to an item did not know how to attack the problem.

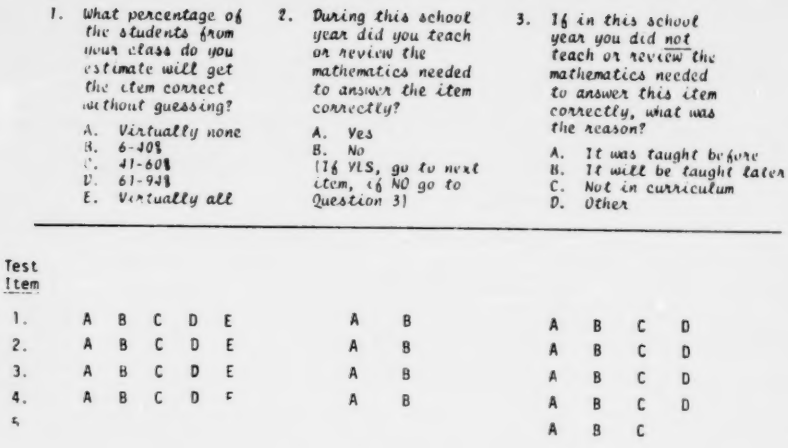
A further consideration in assessing the credibility of scores was the psychometric properties of the items. The basic question investigated was: Did the item in each subtest of the core test tend to measure the same ability, that is, did the students who correctly answered a particular item also do well on the subtest to which the item belonged? As a corollary, did students who incorrectly answered the item do less well on the subtest? On the core pretest, three items were less than satisfactory; on the posttest, just one item posed problems, and that item was also troublesome on the pretest. Nevertheless, a single questionable item out of 40 was considered to be very good.

The rotated forms contained items which, in general, were less relevant to the B. C. curriculum than the core items. As a result, the number of items on the rotated forms showing less than desirable characteristics increased in comparison with the core test. Three or four items on each form were unsatisfactory. Of the total of 14 such items, 11 were very difficult having p-values of less than 25%. Of the five items having the highest non-response rates, four were in the unsatisfactory category. Half of the unsatisfactory items were in the geometry classification. In summary, approximately 10% of the items on the rotated forms showed indications of not being suited to British Columbia students or to the B. C. curriculum.

5.1 Suitability of Items to the B. C. Curriculum

The matter of suitability to the B. C. curriculum was one of particular concern for items on the core test. Performance on these items was to provide a measure of gains over the year, thus the items were subjected to particularly close scrutiny. There were several ways of determining whether the collection of items was suitable for British Columbia schools, some direct, some indirect.

The clearest indicator of item suitability was the "opportunity-to-learn" (OTL) measure. Obtaining these measures called for a considerable effort by the teachers as they were asked to provide information on all test items (40 on the core, 140 on the rotated forms). Figure 5-1 contains a portion of the instrument used to collect the OTL data.



1. What percentage of the students from your class do you estimate will get the item correct without guessing?
   A. Virtually none
   B. 6-40%
   C. 41-60%
   D. 61-94%
   E. Virtually all

2. During this school year did you teach or review the mathematics needed to answer the item correctly?
   A. Yes
   B. No
   (If YES, go to next item, if NO go to Question 3)

3. If in this school year you did not teach or review the mathematics needed to answer this item correctly, what was the reason?
   A. It was taught before
   B. It will be taught later
   C. Not in curriculum
   D. Other

Test Item												
1.	A	B	C	D	E	A	B	A	B	C	D	
2.	A	B	C	D	E	A	B	A	B	C	D	
3.	A	B	C	D	E	A	B	A	B	C	D	
4.	A	B	C	D	E	A	B	A	B	C	D	
5.								A	B	C		

Figure 5-1. A portion of the opportunity-to-learn (OTL) form.

From this information a simple three point scale was constructed. The basic question was whether students had had the opportunity to learn the material in a particular question. Each teacher's response was used to determine whether the mathematics content for an item has been:

- taught or reviewed during the year,
- not taught or reviewed during the year because it had been taught before, or
- not taught or reviewed.

The third category was of primary interest for looking at student achievement, regardless of whether the topic was not in the B. C. curriculum or whether it would be taught in later grades.

Because of a printing error, the OTL measure was not obtained on five items, specifically item 21 on each test form. The OTL information for the core test items is contained in Table 5-1.

The OTL figures were determined for each item by averaging the responses given by the teachers. For example, for item 1, a Measurement item, 16.9% of the teachers felt the item had been taught before, 70.1% taught the material during the year, and 13.0% felt the students had never been taught the mathematics necessary to answer the item. The

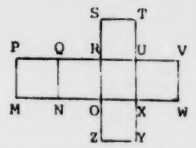
figures shown in Table 5-1 are the means for the items making up each topic.

Table 5-1
Core Items: Mean Opportunity to Learn (%)

Topic	Number of Items	Taught Before	Taught This Year	Never Taught
Algebra	8	1.7	86.5	12.2
Fractions	8	14.4	80.9	4.7
Geometry	7	3.5	51.8	44.7
Measurement	8	15.5	63.0	21.5
Ratio	8	2.9	84.4	12.6
TOTAL	39	7.7	73.9	18.5

The results indicate that the Fraction subtest is particularly well-suited to B. C., that the Algebra and Ratio subtests are reasonably so, that the Measurement subtest may pose some problems, and that the Geometry subtest needs careful scrutiny. In particular, three Geometry items -- 7, 19, 40 -- showed high "never taught" figures: 72.7%, 86.8%, and 63.2%, respectively. On item 19, which asked for the measure of a central angle in a circle given the circumference and length of the subtended arc, 33% of the students correctly answered the question. This figure is considerably better than chance, especially since 22% of the students simply left the question unanswered, the second highest percent non-response on the 40 items. As it happens, this item is one of only two items on the core test which falls into the cognitive behavior category of "analysis", the highest such level tested. Such questions are very difficult to "teach to" since they require students to bring to bear problem-solving skills in addition to lower level knowledge. It may be difficult for a teacher to respond with conviction that the material has been taught, since the necessary individual concepts may have been taught in isolation but not the solution to that particular problem. This problem may be illustrated by considering the Geometry item in the "application" category reproduced in Figure 5-2.

7.



The diagram shows a cardboard cube which has been cut along some edges and folded out flat. If it is folded to again make the cube, which two corners will touch corner P?

A corners Q and S

B corners T and Y

C corners W and Y

D corners T and V

E corners U and Y

Figure 5-2. Core item #7 (Geometry).

In the B. C. curriculum this topic is dealt with at the intermediate level, although it is not core material. However, even though students may not have actually constructed such a three-dimensional shape, if they have a basic familiarity with geometric figures and some imagination, many students should be able to answer the question. In fact 75.9% correctly answered the question and only 4.3% did not attempt it, the lowest non-response rate for any of the Geometry items.

Such considerations led to an examination of the relationship between teacher perceived opportunity to learn and the cognitive level of the items. A summary of these results is shown in Table 5-2.

On four of the five topics on the test the percentage of items rated "never taught" increased with cognitive level. Since the topic of Geometry contained a higher proportion of high level cognitive items than other topics, one might conclude that the seeming incompatibility of the Geometry items with the B. C. curriculum is not as serious as indicated at first glance in Table 5-1.

A second, indirect, indication of appropriateness may be inferred from the non-response data. In the Instructions for Students on each achievement instrument, it was suggested that, if a question was unfamiliar, students should "go on to other questions which are more familiar. Then, if you have time later, you may come back to the questions which you left out."

Table 5-2
Opportunity to Learn, by Cognitive Level

Topic	Cognitive Level	Number of Items	Mean Percent Never Taught
Algebra	I	5	10.5
	II	3	13.0
	III	2	16.0
Fractions	I	3	1.3
	II	3	6.9
	III	2	6.5
Geometry	I	2	22.8
	II	2	22.5
	III	2	68.0
	IV	1	86.8
Measurement	I	2	9.2
	II	3	13.4
	III	2	27.6
	IV	1	58.4
Ratio	I	4	7.8
	II	1	17.1
	II	3	17.7
TOTAL	I	16	9.5
	II	10	13.6
	III	11	26.3
	IV	2	72.6

By phrasing the instructions in this manner it was hoped to reduce the guessing factor, and implicitly to encourage students to leave items blank if they did not know how to do an exercise. Non-response rates averaged across items within topics are presented in Table 5-3.

For all topics the non-response rate decreased from the pretest to the posttest, presumably because more students felt that they possessed some knowledge applicable to the problem. In the areas of Algebra and Geometry the decreases seem natural in that these topics are first dealt with in any extensive way in Grade 8. However, in these two cases, roughly one student in ten failed to respond to each item.

Table 5-3
Core Test Mean Non-response Rates

Topic	Number of Items	Non-response Rate (%) Pretest	Posttest
Algebra	8	18.5	9.5
Fractions	8	6.8	5.5
Geometry	8	17.9	10.3
Measurement	8	5.4	4.2
Ratio	8	7.5	6.1
TOTAL	40	11.2	7.1

No differences were found in the non-response rates at different cognitive levels, indicating that the students were just as willing to tackle a question at the higher levels, such as application, as they were to attempt items which measured computation only.

5.2 Core Test Results

In the discussion which follows, the results for items and for topics are based on the class as the unit of analysis. Since the ultimate aim of the study is to relate student achievement to teaching practice, the basic unit for comparison must be the class average performance on any item. For each item, a class "p-value" was determined. This figure is simply the percentage of students who correctly answered the item out of the total number of students in the class. Class p-values were then averaged across the classes in the study to produce an item p-value. Scores for each topic and for the total test were determined by using the average of the p-values for the items making up each topic.

No correction was made for guessing. Students were not instructed to guess, although there is some evidence to suggest that p-values may be artificially inflated through correct guessing. Four Geometry items on the rotated forms were inspected for their relevance to this problem. On none of the items could it be reasonably expected that a Grade 8 B. C. student would obtain a correct answer. Two items dealt with vectors, and the third with projections. On the fourth item the diagram was missing so the answer was impossible to determine. The results for these items are shown in Table 5-4.

Table 5-4
Achievement on Non-curriculum Items

Item	Topic	Percent Correct	Percent Non Response	Percent Responding	Percent Correct of Respondents
A8	projections	14.8	46.0	54.0	27.4
B11	vectors	13.1	34.9	65.1	20.3
B14	(no diagram)	14.7	25.7	74.3	19.8
D18	vectors	13.5	49.8	50.2	26.9

If one were to assume that the respondents to each item simply guessed at the correct value, the expected percent correct would be 20% since each item had five options. The figures in the right-hand column of Table 5-4 are quite close to this theoretical value. Hence the published p-values should be treated with some caution, particularly for the high difficulty items. Gain scores on the core test should not be appreciably affected since the "guessing" factor is effectively cancelled out when subtracting pretest scores from posttest scores.

Gains in achievement were based on only those students who wrote both the pretest and the posttest, exactly 2000 in all. Results were submitted for 90 classes (2368 students) on the pretest and 87 classes (2180 students) on the posttest. Of the 368 students who wrote the pretest but not the posttest, 91 were accounted for by the loss of three classes. The remaining 277 students, over 10% of the sample, may have moved to different classrooms, dropped out of school, or were absent at the time of the posttest. Similarly, the 180 students who wrote the posttest but not the pretest may have been absent for the pretest or have moved into the class during the year. In both cases the effect of the reduction in the number of students was to increase the scores, as shown in Table 5-5.

On most individual items a similar pattern was indicated, that is, higher performance was achieved for those students writing both the pre- and posttests. A possible explanation for such results is that students who move out of or into a school during the course of a school year tend to perform lower than other students. This conjecture is supported by evidence from the 1977 British Columbia Mathematics Assessment (Robitaille & Sherrill, 1977) in which performance is inversely related to the number of schools attended particularly at the levels of Grade 4 and Grade 8.

Table 5-5
Mean Raw Scores on the Core Test

Topic	Number of Items	Pretest Scores		Posttest Scores	
		All Pretest (n = 2368)	Pre & Post (n = 2000)	All Posttest (n = 2180)	Pre & Post (n = 2000)
Algebra	8	3.1	3.2	4.5	4.5
Fractions	8	3.9	4.0	4.5	4.5
Geometry	8	2.9	3.0	4.2	4.3
Measurement	8	4.2	4.3	4.8	4.8
Ratio	8	3.9	4.0	4.7	4.7
TOTAL	40	18.1	18.4	22.7	22.8

The results on the five topics contained in the core test are discussed in the following pages. Detailed information on each item is presented in graphical form which is described in Figure 5-3. The graph of each item shows the non-response rates on the pre- and posttests, the p-values on the pre- and posttests, the teacher-judged OTL of the item, and the cognitive level of the item.

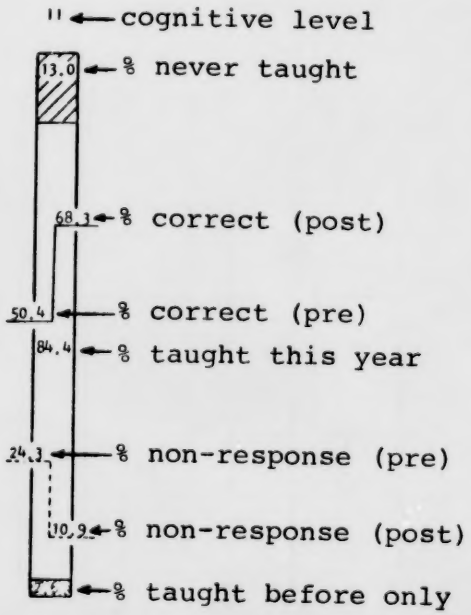

" ← cognitive level
13.0 ← % never taught
68.1 ← % correct (post)
50.4 ← % correct (pre)
84.4 ← % taught this year
24.3 ← % non-response (pre)
10.9 ← % non-response (post)
← % taught before only

Figure 5-3. Interpretation guide to Figures 5-4, 5-6 5-8, 5-10, 5-12.

Algebra

Teachers were nearly unanimous that all of the material related to this topic had to be taught or reviewed during the year. However, the extent of algebraic knowledge possessed by the students leaving the elementary school appears to be quite high, particularly on questions involving the use of variables: items 13, 18, 27, 35.

Pupils entering Grade 8 clearly could not deal with the solution of the equation $5x + 4 = 4x - 31$, nor could they interpret scientific notation with negative exponents. But neither could they do so at the end of the year, in spite of the fact that 80% of the teachers indicated that they had taught this material during the year.

The OTL figures for item 27 are surprising. The item requires the student to set up an open expression to represent the facts in a straight-forward word problem. Over a quarter of the teachers said they did not teach such a procedure, yet over three-quarters of the students responding to the item correctly answered it. It may be that the solution is a matter of common sense, provided that a student understands the basic idea of using letters as variables.

Performance showed improvement from pre- to posttest across all items and non-response rates consistently declined. The greatest improvement was in determining the product of two negative numbers, a procedure that all of the teachers dealt with during the year.

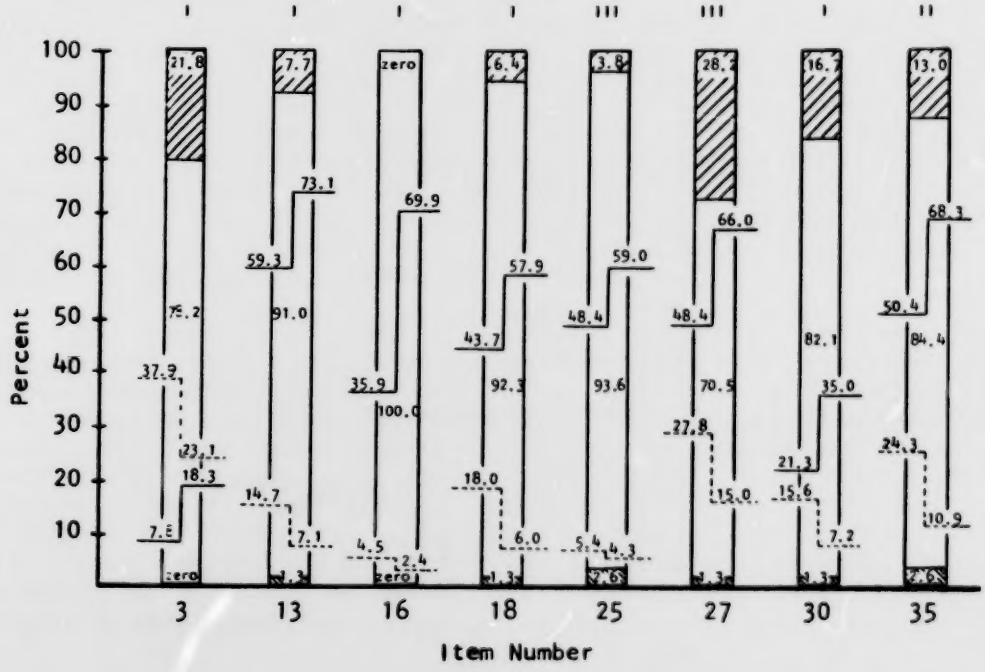


Figure 5-4. Results on the Algebra subtest.

3. If $5x + 4 = 4x - 31$, then x is equal to

   (A.) -35

   B. -27

   C. 3

   D. 27

   E. 35

13. If $P = LW$ and if $P = 12$ and $L = 3$, then W is equal to

   A. $\frac{3}{4}$

   B. 3

   (C.) 4

   D. 12

   E. 36

16. $(-2) \times (-3)$ is equal to

   A. -6

   B. -5

   C. -1

   D. 5

   (E.) 6

18. If $\frac{4x}{12} = 0$, then x is equal to

   (A.) 0

   B. 3

   C. 8

   D. 12

   E. 16

25. The air temperature at the foot of a mountain is 31 degrees. On top of the mountain the temperature is -7 degrees. How much warmer is the air at the foot of the mountain?

   A. -38 degrees

   B. -24 degrees

   C. 7 degrees

   D. 24 degrees

   (E.) 38 degrees

27. A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing $2y$ kg. What weight of tea does he now have?

   A. $x - 15 - 2y$

   B. $x + 15 + 2y$

   (C.) $x - 15 + 2y$

   D. $x + 15 - 2y$

   E. None of these

30. 0.00046 is equal to

   A. 46×10^{-3}

   (B.) 4.6×10^{-4}

   C. 0.46×10^{3}

   D. 4.6×10^{0}

   E. 46×10^{-5}

35. The sentence "a number x decreased by 6 is less than 12" can be written as the inequality

   A. $x - 6 > 12$

   B. $x - 6 \geq 12$

   (C.) $x - 6 < 12$

   D. $6 - x \geq 12$

   E. $6 - x < 12$

Figure 5-5. Algebra items on the core test.

Fractions

An average of about 15% of the teachers felt that the material for the fraction items had been dealt with in previous grades to the extent that it was not necessary to teach or review the material in Grade 8. This appears quite reasonable since the topic of fractions constitutes a large part of the mathematics program in the intermediate grades. Most of the remaining teachers spent some time teaching the topic.

Of particular concern is item 15. This is an estimation item, a topic of increasing importance in dealing with calculators. The most popular option on both the pretest and posttest was 0.00006, indicating that students may be applying a mechanical process rather than dealing with the quantities with understanding. Performance on item 4, a word problem involving the addition of four fractional quantities, proved difficult, with only minor improvement taking place over the year. Performance on the remaining items, most of a simple computational nature, was reasonable, with small but constant gain.

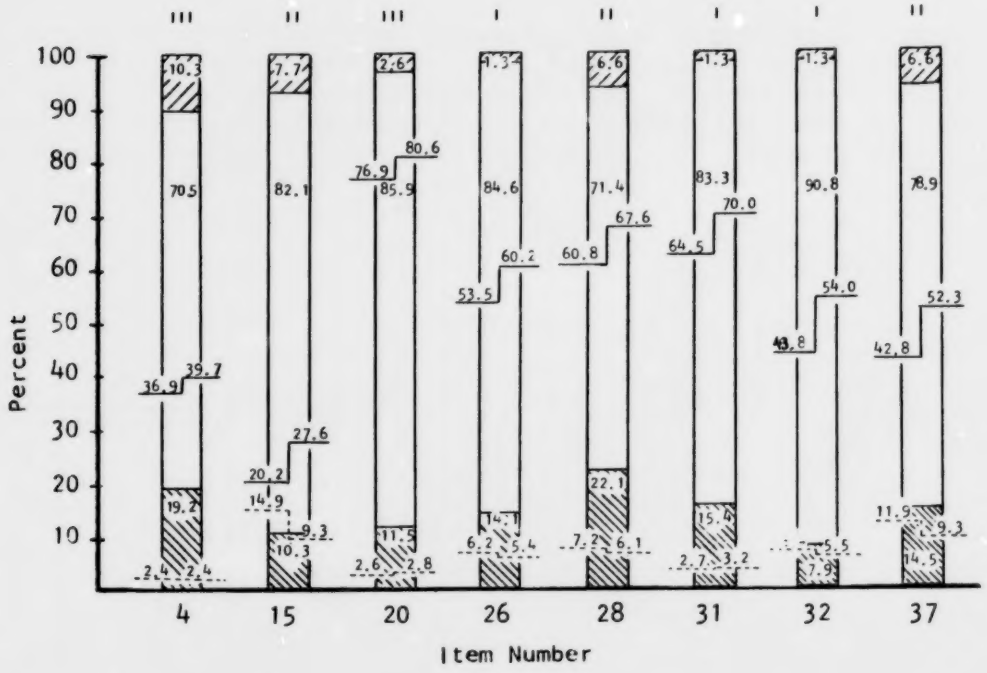


Figure 5-6. Results on the Fractions subtest.

4. Four 1-litre bowls of ice cream were set out at a party. After the party, 1 bowl was empty, 2 were half full, and 1 was three quarters full. How many litres of ice cream had been EATEN?

A. $3\frac{3}{4}$

B. $2\frac{3}{4}$

C. $2\frac{1}{2}$

D. $1\frac{3}{4}$

(E.) None of these

15. The value of 0.2131 x 0.02958 is approximately

A. 0.6

B. 0.06

(C.) 0.006

D. 0.0006

E. 0.00006

20. In a discus-throwing competition, the winning throw was 61.60 metres. The second place throw was 59.72 metres. How much longer was the winning throw than the second place throw?

A. 1.12 metres

(B.) 1.88 metres

C. 1.92 metres

D. 2.12 metres

E. 121.32 metres

26. 0.40 x 6.38 is equal to

A. .2552

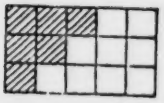
B. 2.452

(C.) 2.552

D. 24.52

E. 25.52

28.



In the figure the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to

A. $\frac{2}{15}$

B. $\frac{1}{3}$

(C.) $\frac{2}{5}$

D. $\frac{3}{8}$

E. $\frac{1}{2}$

31. $\frac{2}{5} + \frac{3}{8}$ is equal to

A. $\frac{5}{13}$

B. $\frac{5}{40}$

C. $\frac{6}{40}$

D. $\frac{16}{15}$

(E.) $\frac{31}{40}$

32. $7\frac{3}{20}$ is equal to

A. 7.03

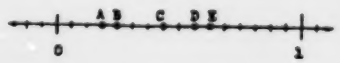
(B.) 7.15

C. 7.23

D. 7.3

E. 7.6

37. Which of the points A, B, C, D, E on this number line corresponds to $\frac{5}{8}$?



A. point A

B. point B

C. point C

D. point D

(E.) point E

Figure 5-7. Fraction items on the core test.

Geometry

The results on this topic show a number of inconsistencies. On item 7 students did the best of the eight items, although about three-quarters of the teachers apparently did not deal with the material. Less dramatic, but still striking is a similar result on item 40. It may be that these items appear technically difficult, for example, in the use of the term "similar" in item 40, but they may provide sufficient clues so that the intent is clear.

The relatively low performance and low gain on item 17 is surprising since the item deals with terminology only, and the topic is specifically required in the Geometry portion of the B. C. syllabus. Considerable gain was achieved on item 39, on which the student was required to identify the coordinates of a point in a plane.

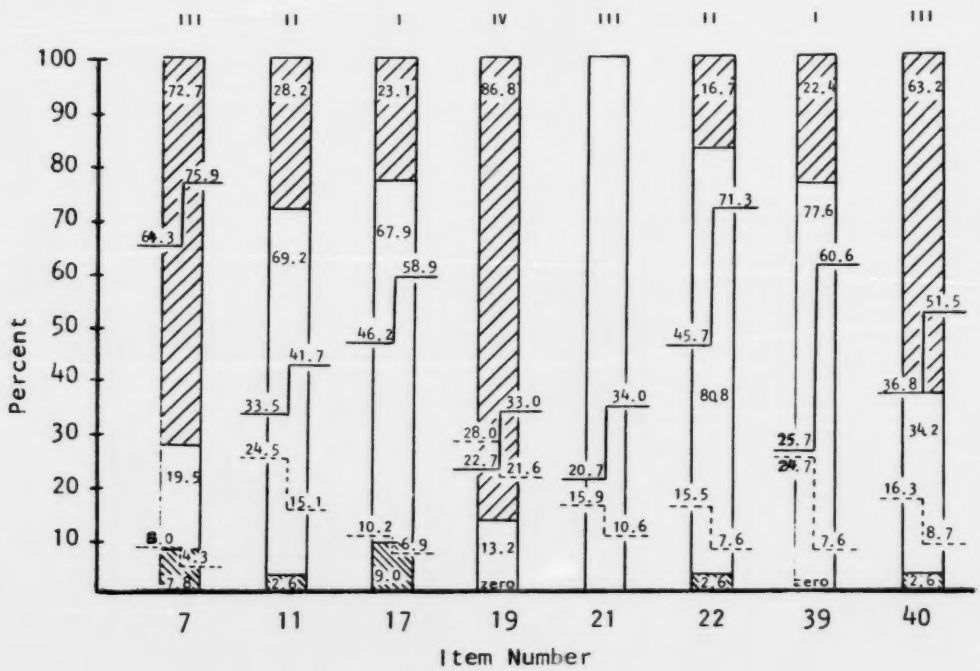
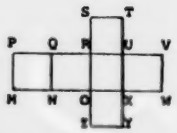


Figure 5-8. Results on the Geometry subtest.

7.



The diagram shows a cardboard cube which has been cut along some edges and folded out flat. If it is folded to again make the cube, which two corners will touch corner P?

A corners Q and S

B corners T and Y

C corners W and Y

(D) corners T and V

E corners U and Y

11. On a number line two points A and B are given. The coordinate of A is -3 and the coordinate of B is +7. What is the coordinate of the point C, if B is the midpoint of the line segment $\overline{AC}$?

A. -13

B. $-\frac{1}{2}$

C. +2

D. +12

(E.) +17

17. Which of the indicated angles is ACUTE?

(A.)

B.

C.

D.

E.

19.

The length of the circumference of the circle with centre at O is 24 and the length of arc RS is 4. What is the measure in degrees of the central angle ROS?

A. 24

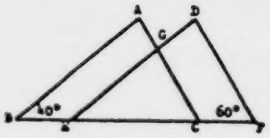
B. 30

C. 45

(D.) 60

E. 90

21.



In the above diagram, triangles ABC and DEF are congruent, with BC = EF. What is the measure of angle EGC?

A. 20°

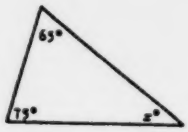
B. 40°

C. 60°

(D.) 80°

E. 100°

22.



x is equal to

A. 75

B. 70

C. 65

D. 60

(E.) 40

39.

What are the coordinates of point P?

A. (-3,4)

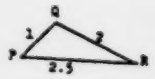
B. (-4,-3)

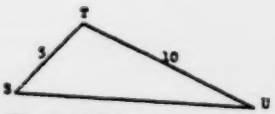
C. (3,4)

(D.) (4,-3)

E. (-4,3)

40





Triangles PQR and STU are similar. How long is SU?

A. 5

B. 10

(C.) 12.5

D. 15

E. 25

Figure 5-9. Geometry items on the core test.

Measurement

On two particular items, the teachers' faith in the efficacy of the elementary school appeared justified. Achievement on items 8 and 29 could hardly be better. On the other hand, for item 9, which required the student to identify the location of an arrow on a scale in a diagram, only a third of students entering Grade 8 were successful and less than half were successful at the end of the grade. This, taken together with the results of item 10, may indicate that there is a need for the use of concrete materials at this grade level. Of the five topics in the core test, measurement is the one most directly related to the "real" world and most amenable to the use of actual measuring devices.

Performance on item 5 is difficult to assess since it is not known whether it is the approximation or the multiplication of 9 x 7 which caused problems.

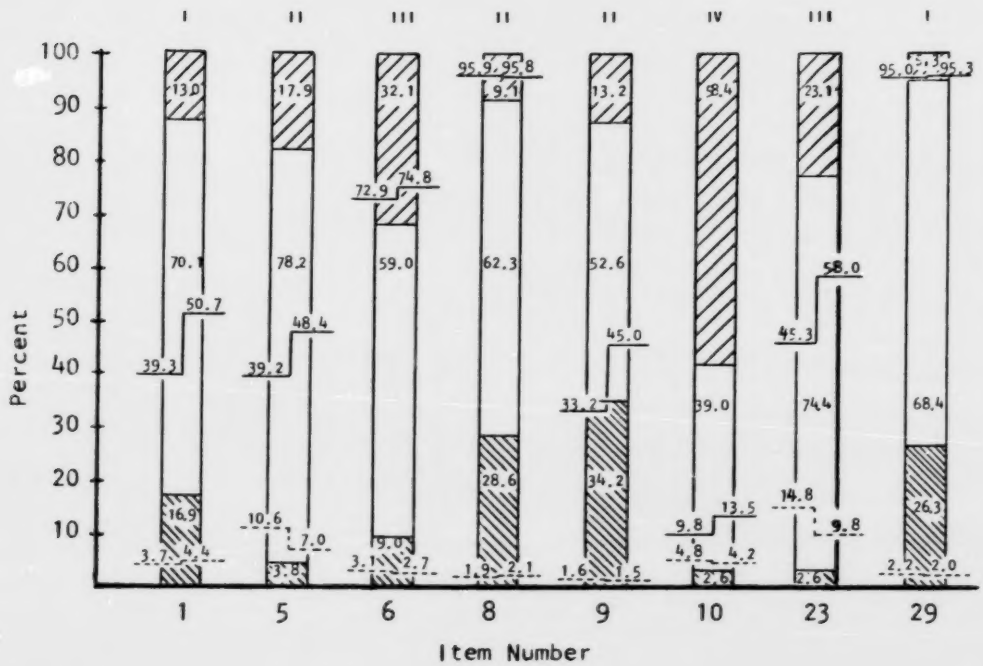


Figure 5-10. Results on the Measurement subtest.

1. 2 metres + 3 millimetres is equal to

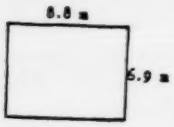
   A. 2.0003 metres

   (B.) 2.003 metres

   C. 2.03 metres

   D. 2.3 metres

   E. 5 metres

5. 

Which of the following is the closest approximation to the area of the rectangle with measurements given?

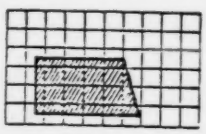
   A. 48 m^2

   B. 54 m^2

   C. 56 m^2

   (D.) 63 m^2

   E. 72 m^2

6. 

□ 1 square unit

The area of the shaded figure, to the nearest square unit, is

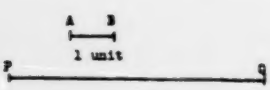
   A. 23 square units

   B. 20 square units

   C. 18 square units

   (D.) 15 square units

   E. 12 square units

8. 

The length of $\overline{AB}$ is 1 unit. Which is the best estimate for the length of $\overline{PQ}$?

   A 2 units

   (B) 6 units

   C 10 units

   D 14 units

   E 18 units

9.

On the above scale the reading indicated by the arrow is between

   A. 51 and 52

   B. 57 and 58

   C. 60 and 62

   D. 62 and 64

   (E.) 64 and 66

10. A solid plastic cube with edges 1 centimetre long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimetres long.

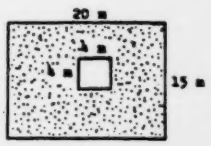
   (A.) 8 grams

   B. 4 grams

   C. 3 grams

   D. 2 grams

   E. 1 gram

23. 

A square is removed from the rectangle as shown. What is the area of the remaining part?

   A. 316 m^2

   B. 300 m^2

   (C.) 284 m^2

   D. 80 m^2

   E. 16 m^2

29. The distance between two towns is usually measured in

   A. millimetres

   B. centimetres

   C. decimetres

   D. metres

   (E.) kilometres

Figure 5-11. Measurement items on the core test.

Ratio

Results on this topic were the most consistent of the five in almost all aspects: teacher OTL assessments, non-response rates, achievement scores, and gain scores. With the exception of item 24, very few teachers assumed that the content had been taught before and did not need to be reviewed. On this topic, as on others, a number of teachers seemed to feel that the mathematics needed to solve word problems was not taught or reviewed, although student performance on such items turned out to be quite high.

Students appear to be reasonably well prepared to deal with ratio, proportion, and percent at the end of elementary school. Only on item 34 was performance lower than might have been expected. Gains in Grade 8 were consistent but moderate.

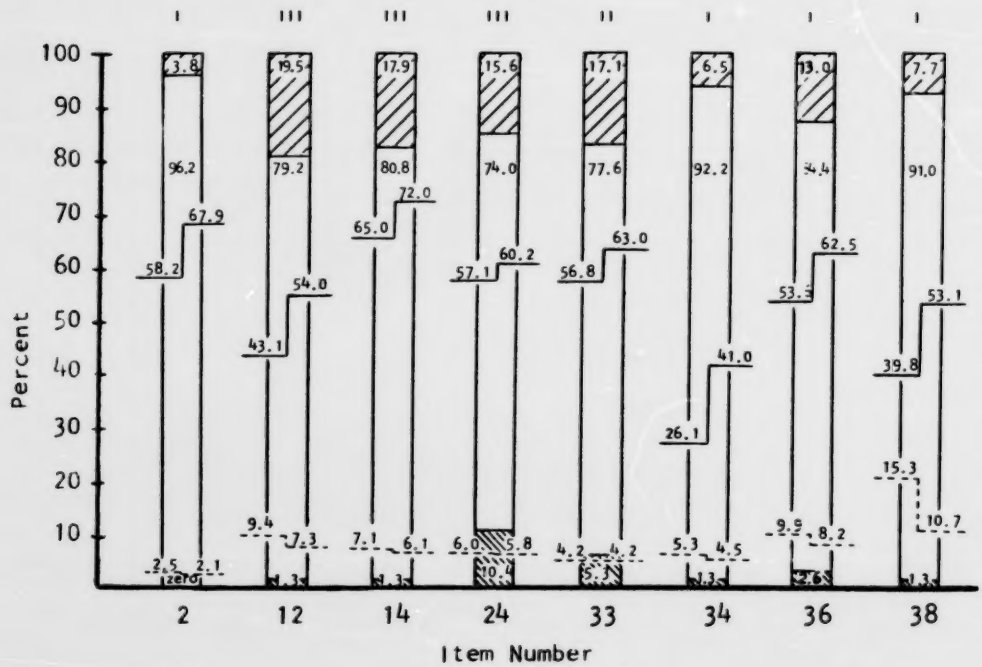


Figure 5-12. Results on the Ratio subtest.

2. $\frac{1}{5}$ =

A. 0.20%

B. 2%

C. 5%

(D.) 20%

E. 25%

12. A painter is to mix green and yellow paint in the ratio of 4 to 7 to obtain the colour he wants. If he has 28 litres of green paint, how many litres of yellow paint should be added?

A. 11

B. 16

C. 28

(D.) 49

E. 196

14. A model boat is built to scale so that it is $\frac{1}{10}$ as long as the original boat. If the width of the original boat is 4 metres, the width of the model should be:

A. 0.1 metre

(B.) 0.4 metres

C. 1 metre

D. 4 metres

E. 40 metres

24. Cloth is sold by the square metre. If 6 square metres of cloth cost $4.80, the cost of 16 square metres will be

(A.) $12.80

B. $14.40

C. $28.80

D. $52.80

E. $128.00

33. In a school of 800 pupils, 300 are boys. The ratio of the number of boys to the number of girls is

A. 3 : 8

B. 5 : 8

C. 3 : 11

D. 5 : 3

(E.) 3 : 5

34. What is 20 as a percent of 80?

A. 4%

B. 20%

(C.) 25%

D. 40%

E. None of these

36. 30 is 75% of what number?

(A.) 40

B. 90

C. 105

D. 225

E. 2250

38. 20% of 125 is equal to

A. 6.25

B. 12.50

C. 15

(D.) 25

E. 50

Figure 5-13. Ratio items on the core test.

5.3 Gains in Achievement

On all of the items, student achievement was better on the posttest than on the pretest. The range of differences of p-values was from virtually no change to a maximum of about 35 percentage points for item 39. The median gain in p-value was 10.4%. Inspection of the distribution of gain scores shows the bulk of scores to be in the range of 5 to 15 percentage points.

Gain scores on three particular items were much greater than the rest, so much so that they must be considered outliers. On item 39, the gain was 34.9 percentage points, the highest of any item. On this question students were asked to identify the coordinates of a given point shown on a Cartesian coordinate plane. This topic is not to be found in the Curriculum Guide for the Grade 8 program (c.f., Curriculum Development Branch, 1978, pp. 22-26), although the material does appear in the authorized textbooks. Since the item involved the use of integers, a topic taught for the first time in Grade 8, one must assume little prior student knowledge. Considering that many of the teachers indicated they did not deal with the topic, the increase in p-value from 26% to 61% is impressive.

The second item showing extraordinary growth was item 16, in which students were required to identify the product of -2 and -3. Seventy percent of the students were correct on the posttest, 36% on the pretest. In each case, the most popular incorrect response was -6: 20% and 47% respectively. Again, although the topic is not part of the elementary school program, over a third of the students entering Grade 8 could correctly answer the question, an indication that some students are being exposed to integers earlier than Grade 8. This particular item is likely dealt with at length by Grade 8 teachers, being one of the basic tasks in the teaching of elementary Algebra.

The third item of high gain was item 22, a geometry question requiring the calculation of the measure of the third angle of a triangle given the measures of the other two angles. This is a topic which appears for the first time in the Grade 8 curriculum. Yet almost half the students correctly answered the question on the pretest and this percentage rose to 71% on the posttest.

In general, growth occurred to a greater extent on the algebraic and geometric items, as is shown in Table 5-6. The mean gain score for the two topics has been influenced by the outliers. A more suitable indication is given by the median gains.

Table 5-6
Gains in p-values

Topic	Mean Gain	Median Gain
Algebra	16.5	14.0
Fractions	6.6	6.8
Geometry	16.9	13.0
Measurement	6.4	6.5
Ratio	9.3	9.3
TOTAL	11.0	10.4

A factor which complicates the assessment of gains is the relationship between pretest scores and gain scores. A scatterplot of the two variables is shown in Figure 5-14.

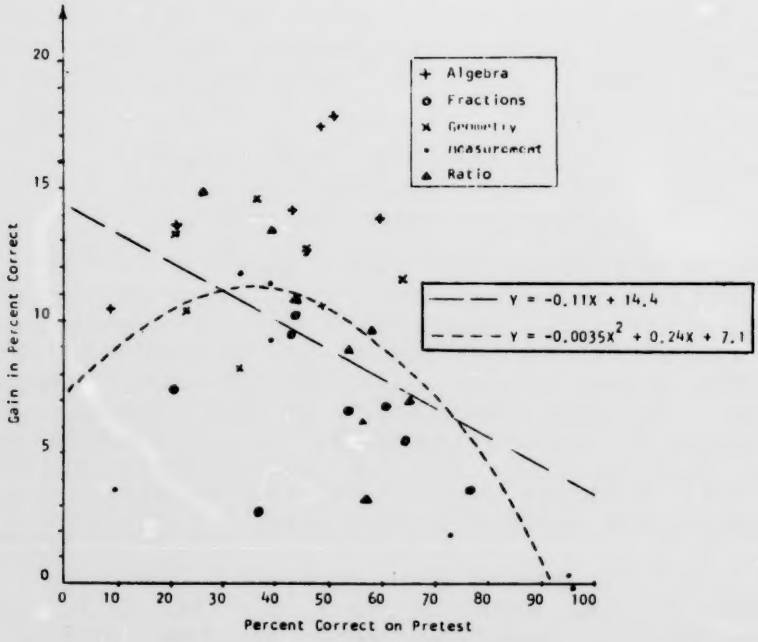


Figure 5-14. The relationship between pretest scores and gains for 37 core test items.

After removing the three outliers, the best fitting line is the second degree curve, considerably better than the simple linear relationship (R^2 quadratic = 0.42; R^2 linear = 0.23). Gain scores tend to decrease as the items become easier; this is the typical "ceiling effect". Little improvement can be made on items which are very easy to begin with. Furthermore, at the other end of the difficulty spectrum, very difficult items tend to show low gain.

The most difficult item on the core test was item 10, but very few students elected not to answer the question. Performance on both pretest and posttest was below the chance level. The item is shown in Figure 5-15.

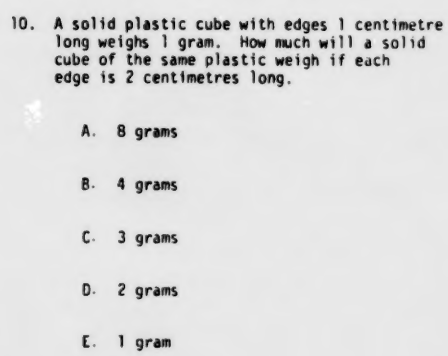

10. A solid plastic cube with edges 1 centimetre long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimetres long.
A. 8 grams
B. 4 grams
C. 3 grams
D. 2 grams
E. 1 gram

Figure 5-15. Core item #10 (Measurement).

This item is difficult for two reasons. It was in the highest cognitive category, analysis, and only about 40% of the teachers indicated they had taught the mathematics to deal with it. About two-thirds of the students selected two grams as the answer. The next most popular response was four grams. One can speculate that the emphasis given in the Curriculum Guide and textbooks to calculating volumes, given the formula, may not be an adequate approach to ensuring that students appreciate the relationships among linear, areal, and volumetric measures.

Item 3 was the second most difficult item, and required the solution of the equation $5x + 4 = 4x - 31$. On both testing occasions, p-values were at the chance level. However, on the posttest there were indications that the better students correctly answered the question.

Items 4 and 15, both on the Fractions subtest, showed low growth coupled with high difficulty. On item 4 the distribution of answers remained virtually unchanged from pretest to posttest. Part of the difficulty may lie in the fact that the option "None of these" was correct, one of the few instances where this was the case. As well, the item is a word problem requiring the interpretation and manipulation of four quantities, not an easy task for a Grade 8 student. On item 15, however, the results are disappointing, both in terms of difficulty and growth. The most popular option was 0.00006. There seems to be little justification for this option, either on mechanical or intuitive grounds, and one must wonder whether students chose it simply because it had the largest number of places to the right of the decimal point.

5.4 Opportunity to Learn and Achievement

One question which may be answered in a preliminary way is whether teachers have reasonable perceptions of students' entry skills coming into Grade 8. Is their assessment of topics upon which to devote their teaching energies valid? In general, if a teacher has made a correct assessment of the curriculum, student performance on the pretest on items for which the teacher had assumed prior knowledge should be higher than on those items which the teachers reviewed or taught during the year.

To explore the issue, correlations were determined between student performance and teacher-assessed opportunity to learn for the items in each topic. Results are shown in Table 5-7.

Table 5-7
Correlation Between OTL (Taught Before) and
Student Achievement on the Pretest

Topic	Number of Items	Correlation, r
Algebra	8	0.61
Fractions	8	0.20
Geometry	7	0.83*
Measurement	8	0.47
Ratio	8	0.29
TOTAL	39	0.46*

* $p < 0.05$

All correlations are positive, and the correlation for all the items on the core test is statistically significant ($p < 0.05$). Nevertheless the overall correlation is not large, and wide variation exists for individual topics. On some particular items students have a lot more knowledge than teachers may give them credit for (e.g., items 2, 13), or they may need a review of material which teachers feel is not necessary (e.g., item 9).

A confounding factor in the preceding discussion, an issue that needs to be clarified in the future with more detailed analysis, is how teachers interpreted the OTL questions. How much time is necessary to spend on a topic in order that it is classified as having been "reviewed or taught"? To some teachers a brief mention may constitute reviewing a topic, while to others a formal review may be implied. Thus, the findings dealing with OTL are suggestive rather than definitive.

The same concerns apply to the use of OTL in the following discussion of gain scores. One might expect if a topic is taught or reviewed during the year that gains in performance should be achieved. Correlations between growth and percentage of teachers who taught or reviewed items on the topics during the year were determined.

All topics showed positive correlations, but only for Ratio was the size of the correlation statistically significant ($p < 0.05$). For the total test, the correlation was small and not statistically significant. One must conclude that there is no relationship or that the measures used are too crude to yield credible results.

A second analysis was carried out to see if growth and OTL were linked when related to specific cognitive levels. The items were grouped in three behavioral categories: computation; comprehension; and application and analysis. It was reasoned that the lower cognitive items might show a connection between OTL and growth, since such items are more amenable to the teaching process. The results across items are shown in Table 5-8.

The figures show a consistent tendency for teachers to indicate that higher cognitive level items were not dealt with during the year, although there is much greater variation in teacher responses for the highest category. Lower growth is indicated as items increase in cognitive complexity. Gain is smaller but more consistent for the upper categories as can be seen from the decreasing standard deviations of gain scores.

Table 5-8
OTL (Taught or Reviewed) and Gains, by Cognitive Level of Items

	Cognitive Level	Number of Items	OTL Mean	OTL S.D.	Gains Mean	Gains S.D.
I.	Computation	16	84.4	9.9	13.4	9.0
II.	Comprehension	10	73.8	10.0	10.3	7.0
III.	Application & Analysis	13	61.1	26.1	8.5	5.1

5.5 Classroom-Based Gain Scores

A second way of examining gains is by considering the classroom as the unit of analysis for each of the topics. These results are summarized in Figure 5-16.

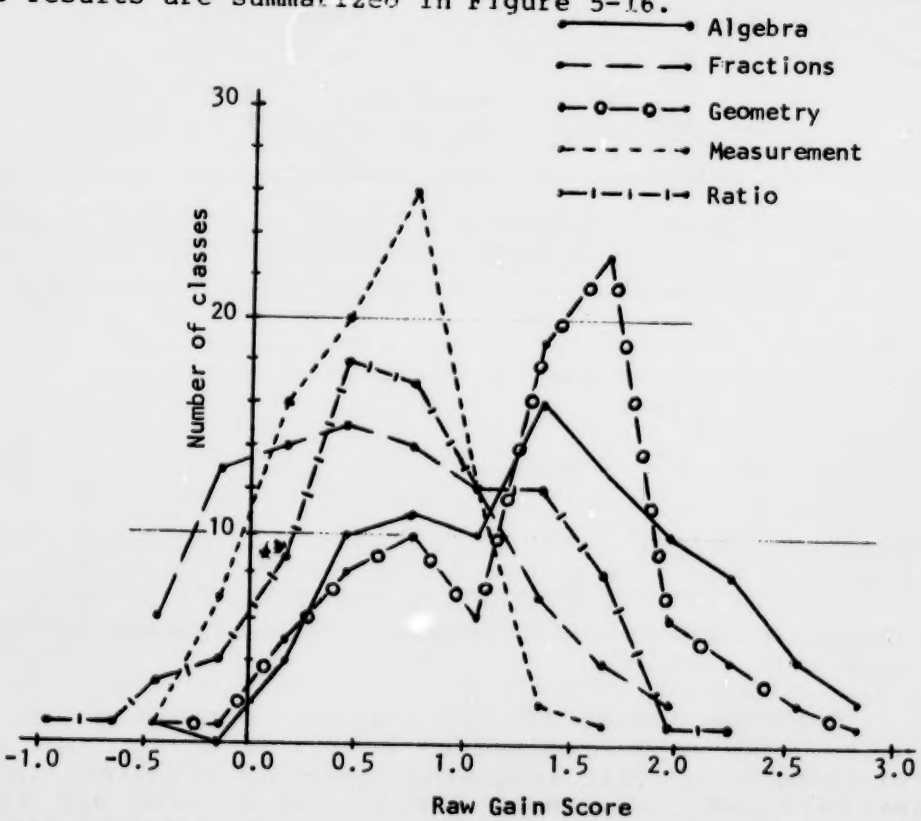


Figure 5-16. Frequency polygons for gain scores on the five core subtests.

Gains tended to be greater for Algebra and Geometry, the two topics dealt with in Grade 8 for the first time on a formal basis. However, gains also showed the widest variation on these two topics. The other three topics tended to have similar gain profiles with Ratio showing somewhat higher gains, and Measurement with the most consistent, but low gains.

The number of negative gains is cause for concern. In these instances, students in a class performed less well at the end of the year than at the beginning. This phenomenon is most evident for the three topics which continue the process of elementary mathematics education: Fractions, Ratio, and Measurement. This may suggest that teachers must review and consolidate work learned in previous years as well as introduce the newer topics of Algebra and Geometry.

The decline in achievement does not seem to be the result of the presence of a few generally ineffective teachers. The distribution of gain and loss across subtopics is presented in Table 5-9. The topics on which loss occurred are also indicated in the table.

Table 5-9
Gain/Loss Achievement Patterns of Classes Across Topics*

5 Gains 0 Losses	4 Gains 1 Loss	3 Gains 2 Losses	2 Gains 3 Losses	1 Gain 4 Losses	0 Gains 5 Losses
60	18	5	3	1	0
	A: 1	F&M: 2	F&M&R: 3	F&G&M&R	
	F: 11	F&R: 2			
	G: 0	G&R: 1			
	M: 4				
	R: 2				

* A=Algebra; F=Fractions; G=Geometry; M=Measurement; R=Ratio

Over two-thirds of the classes improved on all five topics. About 20% gained on four topics but lost on the remaining one. That remaining topic was Fractions in 11 of the 18 cases. In about 10% of the classes was there a decreased performance on two or more topics.

The results were also examined to determine whether the losses tended to occur in classes where the pretest means were high. In particular, the topic of Fractions was examined for regression to the mean effect, since the highest number of losses was recorded for this topic. There was found to be no significant correlation ($p < 0.05$) between pretest score and gain score. Negative gain scores were shown at all levels of pretest performance, adding weight to the conjecture that practice and review are necessary to maintain skill levels on topics thought to be covered in the earlier years of school.

5.6 The Rotated Forms

For several reasons, results on the rotated forms are more difficult to assess than those on the core test. The forms were administered to provide a wide coverage of the curriculum in order to arrive at valid international comparisons. Since B. C. was one of the first "countries" to administer the tests, no international information is available at this time for comparative purposes. Students were tested only once on these items, at the end of the year, so no gain scores are available. Finally, since interpretation panels to decide upon the adequacy of performance were not part of the project, it is difficult to assess the level of performance by item or topic.

The topics dealt with on the rotated forms were the same as those on the core test, with additional items under the headings of Arithmetic, and Probability and Statistics. The items, along with their p-values, are grouped by topic in Appendix F. In general, the items were more difficult, students tended to leave them unanswered more often, and teachers claimed less student exposure to the mathematics required to answer the items. These conclusions are based upon the information contained in Table 5-10.

The items on the core topics show a high degree of consistency of difficulty across subtests. On the rotated forms, items which were not included under the core topics tended to be easier. The mean p-value for Probability and Statistics is the highest of any set of items. This group also indicated the highest "never taught" proportion. This seems to be an anomaly until the items themselves are examined. Most items in this group centered on the ability to read data from graphical displays. Although this is not part of the Grade 7 or Grade 8 curriculum, it is a main strand in Grades 4 to 6.

Table 5-10
Comparison of Items on Rotated Forms and Core Test

Topic	Form	Number of Items	Mean p-Value	Mean percent Non-response	Mean percent Never taught
Algebra	Rotated	24	47.0	12.0	19.5
	Core	8	55.9	9.5	12.2
Arithmetic	Rotated	20	60.5	7.0	20.9
	Core	nil	---	---	---
Fractions	Rotated	17	57.8	5.5	8.9
	Core	8	56.5	5.5	4.7
Geometry	Rotated	33	41.5	16.2	52.1
	Core	8	56.1	10.3	44.7
Probability & Statistics	Rotated	18	61.0	8.4	52.7
	Core	nil	---	---	---
Ratio	Rotated	9	46.0	7.4	14.8
	Core	8	59.2	6.1	12.6
TOTAL	Rotated	140	51.0	10.4	30.1
	Core	40	57.6	7.1	23.1

Performance in Arithmetic was also high. These items covered a wide variety of problems in which the common link was the use of whole numbers, including such topics as exponents, square roots, number theory, and patterning.

5.7 Summary

At the Grade 8 level, most items on the core test were compatible with the B. C. curriculum. Non-response rates generally were low. With the exception of several Geometry and Measurement items, most teachers taught the material during the year. Students showed gains in achievement consistently across items and topics. Performance on items at the lower cognitive levels was superior to that at higher levels. Analysis of classroom-based gain scores showed a need for teachers to ensure that students maintain skill levels, particularly on the topic of Fractions

References

Robitaille, D.F., & Sherrill, J.M. British Columbia Mathematics Assessment, 1977: Test Results. Victoria, B.C.: Ministry of Education, 1977.

Curriculum Development Branch. Mathematics Curriculum Guide, Year One to Twelve. Victoria, B.C.: Ministry of Education, 1978.

CHAPTER 6

STUDENTS' ACHIEVEMENT: GRADE 12

The achievement items were distributed among eight "rotated" forms. In contrast to the Grade 8 tests, there was no core form to which all Grade 12 students responded, nor was any pretest administered. The cognitive results are based solely on tests written at the end of the year or semester. Furthermore, because only two or three students in each class responded to any particular form, the results reported here are the means of individual student scores rather than the means of class scores as reported for Grade 8. The effect is to make the student the unit of analysis rather than the teacher, a disadvantage in further correlational analysis.

All items were expressed in multiple choice format with five choices per item. In administering the tests, teachers were asked to make it clear to students that some items might not be familiar to them and that they should not spend time on such items. The use of calculators, rulers, and protractors was not permitted.

Each form contained 17 items, and forms were prearranged so that, in a typical class of 24 students, three students wrote each form. Students were given 45 minutes to complete the test. Although many items were difficult and others required considerable time to identify the correct response, there was no indication that the time was insufficient for any test.

Student and teacher cooperation were excellent. Out of a designed sample consisting of 105 classes, achievement tests were returned for 95 classes containing a total of 1945 students. The results on the eight forms are shown in Table 6-1. Because of a keypunch error in the answer key, the starred figure for Form 3 is an estimate only. The similarity of means and standard deviations suggests that the forms were equivalent in difficulty.

The generally low scores are due to the fact that many of the items dealt with material which is not contained in the B. C. curriculum. Across the countries involved in the IEA study there are wide differences both in the breadth and depth of mathematical topics in the final year of secondary school. To obtain representative achievement information it was necessary to formulate items which were suitable for some countries but clearly not for others. There likely are few items for which all countries would be satisfied.

Table 6-1
Results by Form, Grade 12

Form	Number of Responses	Mean (Out of 17)	Standard Deviation
1	243	5.77	2.59
2	248	6.20	2.58
3	241	5.24	2.30*
4	247	6.15	2.85
5	249	5.15	2.44
6	242	4.95	2.62
7	241	4.95	2.63
8	234	5.00	2.57

* estimated

To make the B. C. interpretation realistic in light of the B. C. curriculum, the 136 items on the eight rotated forms were examined for their local curriculum validity. Items which were explicitly or implicitly included in the Curriculum Guide, either as core, enriched, or optional material, were classified so as to produce meaningful categories containing roughly the same number of items in each. A total of 91 items were identified as relevant, divided among eight reporting categories.

Three factors were used to judge whether the items were indeed suitable for Grade 12 students in British Columbia. The first was the non-response rate. For the 91 items judged relevant, the mean non-response rate was about 16%; for the non-curriculum items the figure was about 45%. Typical results for a non-curriculum item are shown in Figure 6-1. About 60% of the students simply left this item unanswered; the remaining students likely guessed, since the expected response rate for each option based on random selection by the 40% of students who selected an option is about 8%.

11. In the plane E, T_v is the translation corresponding to the vector v, S_Ω is the half-turn about the center Ω, and $h(\Omega,k)$ is the size transformation of magnitude k and center Ω. If

$$f = h(\Omega,3) \circ T_{\vec{i}-\vec{j}} \circ T_{\vec{j}+2\vec{i}} \circ h(\Omega,-\tfrac{1}{3}) \circ S_\Omega$$

then f is

%		
4.5	A	a half-turn about Ω
10.3	B	a half-turn about a point other than Ω
8.6 C	C	a translation
9.5	D	a size transformation of magnitude -1
7.8	E	A reflection in a line through Ω
59.3	Non-response	

Figure 6-1. Example of response rates to a non-curriculum item (Form 1, item 11).

The second factor used to assess the relevance of items to the B. C. curriculum was the p-value. For the curriculum items the mean p-value was about 42%; for the non-curriculum items, about 12%. Given the non-response rate for the latter of about 45%, the 12% figure is close to the random response level.

Finally, the opportunity to learn (OTL) figures were examined. OTL ratings were obtained from Grade 12 teachers in the same fashion as for the Grade 8 teachers. For the curriculum items, the median percent of teachers who stated that the item had never been taught was about 20%, for the remaining items the figure was 93%. In this instance the median has been chosen since the mean was severely affected by the presence of a few extreme cases.

On the basis of these three criteria it was concluded that the general classification of items was correct although, as will be shown in detail, some individual items were likely classified incorrectly, and one entire topic which was thought to be part of the curriculum, is not.

6.1 Non-curriculum Items

Calculus, differential and integral, is the major topic which is not part of the B. C. curriculum but which is studied in the secondary schools of most of the countries in the IEA study. The B. C. data show some indication that a few teachers have dealt with limits and, perhaps, simple differentiation, but for all intents and purposes, the topic is not part of the B. C. curriculum.

The other major difference between the B. C. curriculum and that of some other countries is transformational geometry. Particularly in France and Belgium, this topic is studied extensively and a typical example is that previously shown in Figure 6-1.

The items were classified before classroom results were obtained. In looking at the results some anomalies in classification were apparent. In general, the non-response rates for the non-curriculum items were high (about 45%) and the percentage of correct answers was low (about 12%). However, responses to several items deviated from the norm, as is shown in the example in Figure 6-2.

15. At which point does the curve $y = 3x^2 - x^3$ have a local minimum?

%		
4.4	A	(2,4)
16.1	B	(3,0)
5.2	C	(1,2)
8.5	D	(0,3)
33.5 C	E	(0,0)
32.3	Non-response	

Figure 6-2. Example of a non-curriculum item (Form 2, item 15) showing a high success rate.

The usual method of solution would be to differentiate, set the result equal to zero, and solve for x. However, by misinterpreting the phrase "local minimum", the correct answer (0, 0) might have been guessed by simply selecting the point with the combined lowest x and y values.

For item 16 on Form 3 (see Figure 6-3), presumably the student was expected to set the derivative to zero and solve for r. However, by sketching the cross-section the students may see that the condition for r and h is 2r + h = 3. Knowing this, the first three options are impossible. On the B. C. results the final two options received a higher percentage of responses than the first three.

16.

The intersection of a cylinder with plane through its axis is a rectangle of perimeter 6 m. The radius of the cylinder satisfying this condition and having maximum volume is

%		
11.2	A	2.5 m
12.4	B	2 m
13.7	C	1.5 m
22.0 C	D	1 m
17.4	E	0.5 m
23.2	Non-response	

Figure 6-3. Example of a non-curriculum item (Form 3, item 16) on which success is enhanced by item design.

A similar situation exists for Form 7, Item 11, shown in Figure 6-4.

11. The curve defined by $y = x^3 - ax + b$ has a relative minimum point at (2,3). The values of a and b are

%		
20.7	A	$a = 8\frac{1}{2}$ and $b = 12$
8.7	B	$a = 12$ and $b = 8\frac{1}{2}$
27.8 C	C	$a = 12$ and $b = 19$
2.9	D	$a = 19$ and $b = 12$
2.1	E	$a = 19$ and $b = 32$
37.8	Non-response	

Figure 6-4. Example of a non-curriculum item (Form 7, item 11) on which success is enhanced by intelligent guessing.

The fact that point (2, 3) lies on the curve leads to the condition that $2a - b = 5$. Immediately options B, D, and E are eliminated. As can be seen in Figure 6-4, options A and C received the highest percentage of responses.

In summary, from the evidence provided by non-response rates, p-values, and OTL estimates, the classification of non-curriculum items seems to have been correct.

6.2 Curriculum Items

The items felt to be part of the B. C. curriculum were divided into the categories shown in Table 6-2.

Table 6-2
Classification of Items at the Grade 12 Level

Topic	Number of Items
Sets, Properties, and Numerical Operations	12
Algebra: Exponents, Logarithms, Radicals	10
Algebra: Polynomials and Rational Expressions	8
Algebra: Equations and Inequalities	12
Functions	16
Analytic Geometry and Vectors	10
Trigonometry and Complex Numbers	14
Permutations, Combinations, Probability, Statistics	9
TOTAL	91

Results are presented on the following pages in graphical form for each of the eight sets of items. The figures in the graphs may be interpreted by consulting Figure 6-5. For each item the following information is shown: cognitive level, teacher-judged opportunity to learn ratings, percent correct, and the percent of students who did not respond.

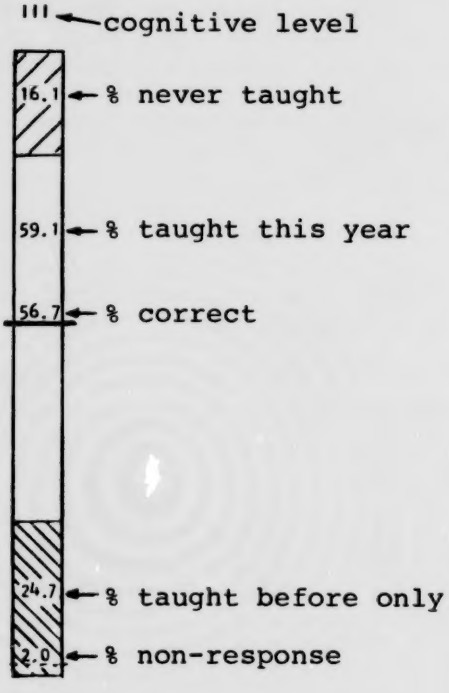


Figure 6-5. Interpretation guide to Figures 6-6, 6-8, 6-10, 6-12, 6-14, 6-16, 6-18, 6-20.

Sets, Properties, and Numerical Operations

This group of items consisted of questions dealing primarily with numerical operations. Teachers assumed that much of the material had been taught in the years before Grade 12. One notable exception was Form 3, item 12, for which the non-response rates, OTL, and percent correct figures indicated that the topic of matrices, an optional topic within the enriched format of the Algebra syllabus, was not taught.

Performance on Form 3, item 7, was low. This item required students to apply the technique of mathematical induction, a procedure which teachers indicated was not extensively taught. The results on Form 2, item 16, indicated that students did not possess a real understanding of the commutative principle. The low level of achievement on Form 5, item 1, a measure of students' understanding of the concept of least common multiples, was surprising, since this topic is introduced at the elementary school level.

On the remaining eight items in this group the OTL figures and consistency of performance, in the range of 50% correct, indicate that the items constituted a reasonably well-defined group. Achievement levels were not high, but as shown by the low percentage of teachers who taught or reviewed the material during the year, not much attention was paid to the topic by the Grade 12 teachers.

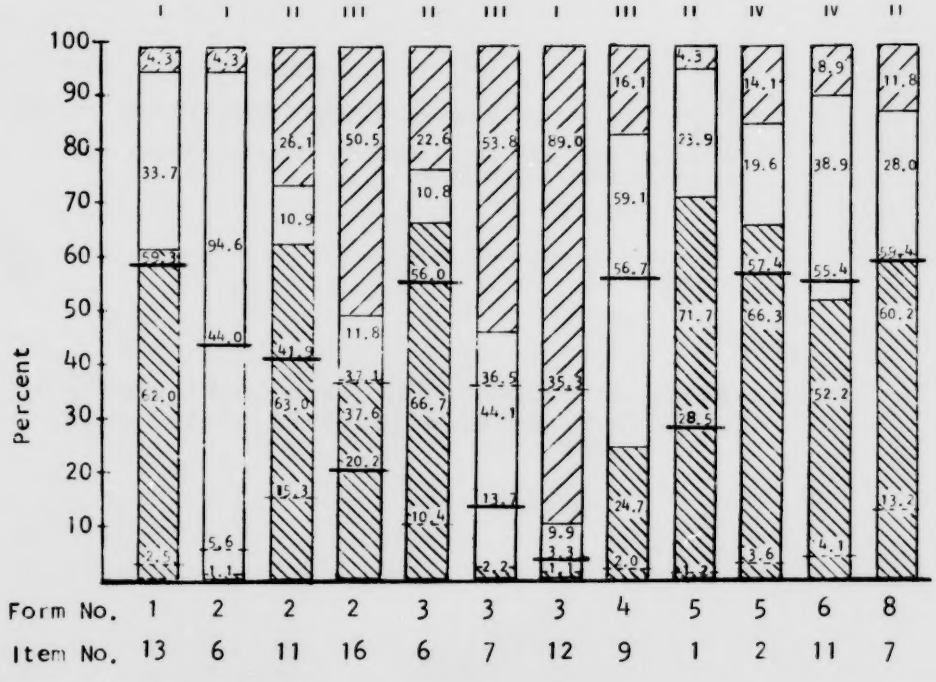


Figure 6-6. Results on Sets, properties, and Numerical Operations.

1-13. Given that $\sqrt{600}$ is approximately 24.4949, an approximation to 2 SIGNIFICANT DIGITS for $\sqrt{600}$ is:

(A) 24
B 24.49
C 24.5
D 24.50
E 25

2-6. The sum of the infinite geometric series

$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$ is

A $\frac{5}{8}$
(B) $\frac{2}{3}$
C $\frac{3}{5}$
D $\frac{3}{2}$
E ∞

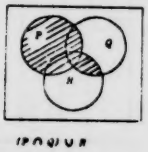
2-11. If X and Y are sets, then $(X \cup Y) \cap (X \cap Y)$ is equal to

A X
B Y
C $X \cup Y$
(D) $X \cap Y$
E $(X \cup Y) \cup (X \cap Y)$

2-16. An operation $*$ (on the set of real numbers) is commutative if, for every two real numbers x and y, $x * y = y * x$. Which of the operations defined below is commutative?

A $x * y = x + xy$
B $x * y = x - y$
C $x * y = x(x + y)$
(D) $x * y = xy(x + y)$
E $x * y = x^2 + xy^2 + y^3$

3-6. The symbol $P \cap Q$ represents the intersection of sets P and Q and the symbol $P \cup Q$ represents the union of sets P and Q. Which of the following represents the shaded portion of the diagram below?



A $(P \cap Q) \cup R$
(B) $P \cup (Q \cap R)$
C $P \cap (Q \cup R)$
D $(P \cap Q) \cap R$
E $(P \cup Q) \cap R$

3-7. If $a_1 = 1$ and $a_{n+1} = a_n + 2n + 1$, then for all n

A $a_n = 4$
B $a_n = 4n + 2$
C $a_n = 2n - 1$
D $a_n = 2n + 2$
(E) $a_n = n^2$

3-12. x and y are real numbers. The product of the matrices

$A = \begin{pmatrix} 1 & x \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ y & 1 \end{pmatrix}$

is commutative only if

A $x = 0$
B $y = 0$
C $x = y$
(D) $x = 0$ or $y = 0$
E $x = 0$ and $y = 0$

4-9. The speed of an object increases uniformly at a rate of 10% per second. At a given time, its speed is 10.00 metres per second. Three seconds later, its speed will be

A 11.10 metres per second
B 12.10 metres per second
C 13.00 metres per second
(D) 13.31 metres per second
E 40.00 metres per second

5-1. If $a = 2^3 \cdot 5^2 \cdot 7$ and $b = 3^2 \cdot 5^3 \cdot 7^2 \cdot 11$ then the least common multiple (LCM) of a and b is

A $5^2 \cdot 7$
B $2^3 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11$
(C) $2^3 \cdot 3^2 \cdot 5^3 \cdot 7^2 \cdot 11$
D $2^3 \cdot 3^2 \cdot 5^5 \cdot 7^3 \cdot 11$
E $2^3 \cdot 3^2 \cdot 5^6 \cdot 7^2 \cdot 11$

5-2. According to one plan for travelling to Mars, the round trip would take almost exactly 3 years, including a stay on Mars of 449 Earth days. If one must travel 34,000,000 miles each way on the trip, which of the following can be used to estimate the average speed of travel in miles per hour?

A $\frac{(3 \times 365 - 449) \times 24}{34{,}000{,}000}$
B $\frac{(3 \times 365 - 449) \times 24}{34{,}000{,}000 \times 2}$
C $\frac{34{,}000{,}000}{(3 \times 365 - 449) \times 24}$
D $\frac{34{,}000{,}000 \times 24}{2 \times (3 \times 365 - 449)}$
(E) $\frac{2 \times 34{,}000{,}000}{(3 \times 365 - 449) \times 24}$

6-11. If a is a digit, let $.\overline{a}$ be the number with decimal expansion $.aaaaaa\ldots$. What is

$.\overline{7} \times .\overline{4}\ 7$

A 1.1
B 1.$\overline{7}$
C 1.2
(D) 1.$\overline{2}$
E 1.3

8-7. A number is the multiplicative inverse of another number if the product of the two numbers is 1. Which of the following sets of numbers is identical to the set of its multiplicative inverses?

A $(1,2,3)$
B $(1,\frac{1}{2})$
(C) $(1,2,\frac{1}{2})$
D $(2,3,5,\frac{1}{2},\frac{1}{3})$
E $(2,3,\frac{2}{3})$

Figure 6-7. Sets, Properties, and Numerical Operations items.

Algegra: Exponents, Logarithms, and Radicals

Of the eight sets of items at the Grade 12 level, the greatest consistency in OTL and non-response rates was demonstrated on this topic. Approximately 90% of the teachers taught the material for each item, and non-response rates averaged about 7%.

Performance which might be of concern is that on Form 7, item 1. The low proportion of correct responses is due in part to the presence of a very attractive distractor (B), which about half of the students selected as the correct response. This item might serve as a particularly good example through which teachers could discuss in detail the restrictions which exist in the real number system.

The results for Form 2, item 17 and Form 3, item 8 indicated that students were able to rationalize the denominator of irrational fractions.

Although almost all of the teachers taught logarithms during the year, only about half of the students were successful on Form 8, items 1 and 15, the two items dealing with logarithms.

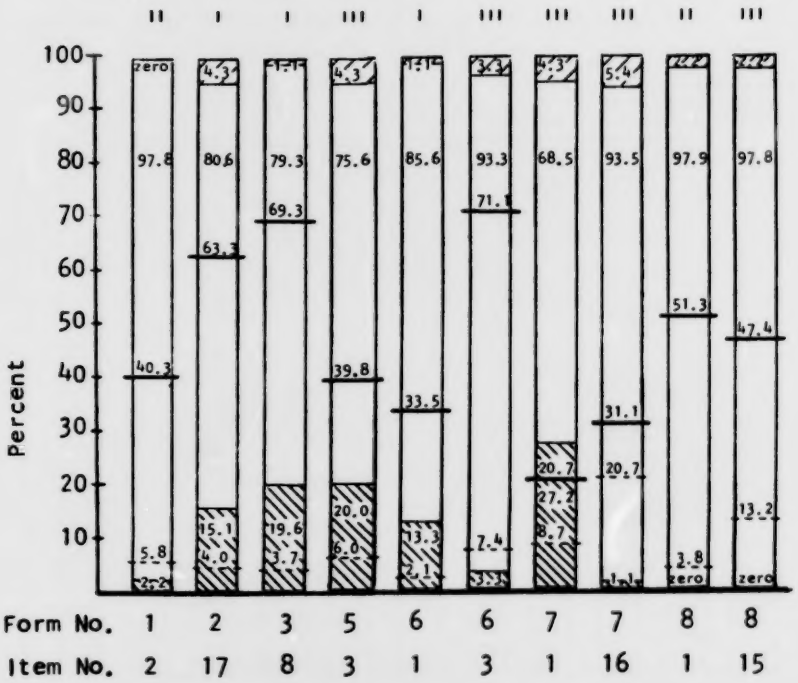


Figure 6-8. Results on Algebra: Exponents, Logarithms and Radicals.

1-2. Given that $a > 0$, $\sqrt[5]{a^2}$ is equal to

(A) $a^{\frac{2}{5}}$

B $a^{\frac{5}{2}}$

C $\frac{1}{\sqrt[5]{a^2}}$

D $\sqrt[3]{a^2}$

E $\sqrt[5]{a}\sqrt[5]{a}$

2-17. If $x > 0$, $y > 0$, and $x \neq y$, then $\frac{1}{\sqrt{x} - \sqrt{y}}$ is equal to

(A) $\frac{\sqrt{x} + \sqrt{y}}{x - y}$

B $\frac{\sqrt{x} - \sqrt{y}}{x - y}$

C $\frac{1}{\sqrt{x} - \sqrt{y}}$

D $\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{y}}$

E $\frac{\sqrt{x} + \sqrt{y}}{x^2 - y^2}$

3-8. $\frac{8}{3 - \sqrt{2}}$ is equal to

A $\frac{8(3 + \sqrt{2})}{5}$

(B) $\frac{8(3 + \sqrt{2})}{7}$

C $\frac{8(3 + \sqrt{2})}{11}$

D $\frac{8(3 + \sqrt{2})}{13}$

E $\frac{8(3 - \sqrt{2})}{13}$

5-3. If x is a real number, then y defined by $y = \sqrt{x^2 - 1}$, is a real number for

A All real values of x

B $x = 0$

C All non-negative real values of x

(D) $x \leq -1$ or $x \geq 1$

E $-1 \leq x \leq 1$

6-1. $3x^5 \cdot 3x^{-5} =$

A $9x^{-5}$

B $9x$

C 0

D 1

(E) 9

6-3. If $3^{x+y} = 81$ and $25^{\frac{x}{y}} = 5$, then y is

A 0

B 2

(C) 3

D $\frac{7}{2}$

E $\frac{15}{4}$

7-1. If x and y are real numbers, for which x can you define y by

$$y = \frac{x}{\sqrt{9 - x^2}}$$

A All x except $x = 3$

B All x except $x = 3$ and $x = -3$

C $x < -3$ or $x > 3$

(D) $-3 < x < 3$

E $x < 3$

7-16. If $10^a = 4$, then 10^{1+2a} is equal to

A 26

B 40

(C) 160

D 900

E 10^3

8-1. If $\log N = n$, then $\log N^2$ is equal to

A $n + 2$

B n^2

C $\frac{n}{2}$

(D) $2n$

E $n - 2$

8-15. Given $\log_b 2 = \frac{1}{3}$, $\log_b 32$ is equal to

A 2

B 5

C $-\frac{3}{5}$

(D) $\frac{5}{3}$

E $\frac{3}{\log_2 32}$

Figure 6-9. Algebra: Exponents, Logarithms, and Radical items.

Algebra: Polynomials and Rational Expressions

Each of the two items showing relatively high non-response rates, Form 4, item 12 and Form 5, item 11, required the analysis of fractional polynomials. This may indicate a reluctance on the part of students to attempt items requiring considerable algebraic manipulation. The reader is invited to test this hypothesis by attempting Form 5, item 11.

Performance on Form 3, item 10 was extremely poor, below the random guessing level. It is highly likely that students used the misleading clue of $(P + Q)(P - Q)$ to arrive at the difference of squares, a distractor chosen by over half of the students.

Although about two-thirds of the students correctly answered Form 8, item 4, this result is not encouraging. Students should be able to identify immediately the fact that the expression is not defined when the denominator is zero. Furthermore, a knowledge of factoring the difference of squares is not required; the correct answer may be obtained by substituting the given values in the denominator.

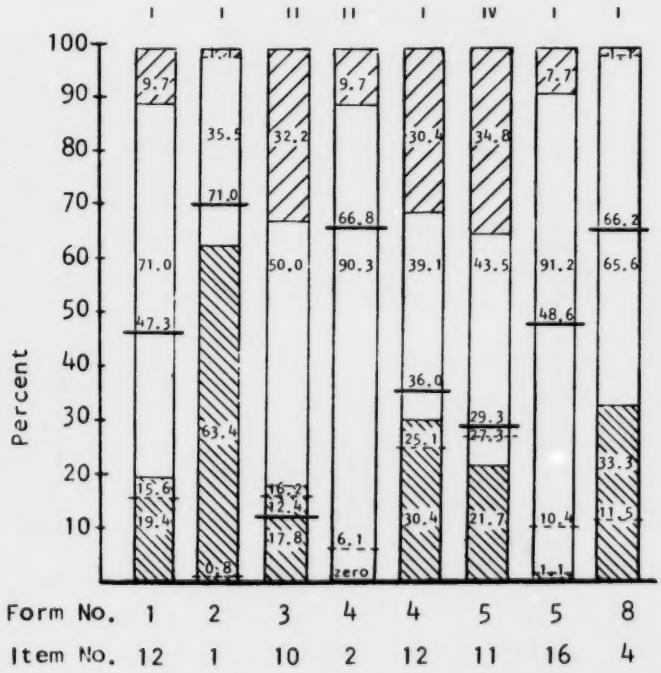


Figure 6-10. Results on Algebra: Polynomials and and Rational Expressions.

1-12. If $2x^2 - 12x + 9 = 2(x - a)^2 + b$, then

A $a = -3$ and $b = -9$

(B) $a = 3$ and $b = -9$

C $a = -3$ and $b = 9$

D $a = -3$ and $b = 27$

E $a = 3$ and $b = 27$

2-1. For all rational numbers a, b, c and d,

$a - (b + (a - d))$

is equal to

A $a - b + a - d$

(B) $a - b - a + d$

C $a - b - a - d$

D $a - b + a + d$

E None of these

3-10. P is a polynomial in x of degree m, and Q is a polynomial in x of degree n, with $n < m$. The degree of polynomial $(P + Q)(P - Q)$ is

(A) $2m$

B m^2

C mn

D n^2

E $m^2 - n^2$

4-2. Which of the following, $(x - 1)$, $(x - 2)$, $(x + 2)$, $(x - 4)$ are factors of $x^3 - 4x^2 - x + 4$?

A Only $(x - 1)$

B Only $(x - 1)$ and $(x + 2)$

C Only $(x - 2)$ and $(x + 2)$

D Only $(x + 2)$ and $(x - 4)$

(E) Only $(x - 1)$ and $(x - 4)$

4-12. If $\frac{x - 1}{x^2 + 3x + 2} = \frac{P}{x + 1} + \frac{Q}{x + 2}$ then

(A) $P = -2$, $Q = 3$

B $P = -3$, $Q = 2$

C $P = -1$, $Q = -2$

D $P = 2$, $Q = -3$

E $P = 3$, $Q = -2$

5-11. The function f, defined by $f(x) = \frac{(x - 1)(3x + 1)}{(2x - 1)(x - 2)}$ is negative for all x such that

A $-\frac{1}{3} < x < 3$

B $\frac{1}{2} < x < 2$

C $1 < x < 3$

D $\frac{1}{2} < x < 2$ or $2 < x < 3$

(E) $-\frac{1}{3} < x < \frac{1}{2}$ or $1 < x < 2$

5-16. When $(1 + p)^6$ is expanded, the coefficient of p^3 is

A 6

B 10

(C) 15

D 20

E 30

8-4. $P(n) = \frac{n^3 - 2n^2 - n + 2}{n^2 - 1}$

$P(n)$ is not defined for

A $n = 1$ and $n = 2$

B $n = -1$ and $n = 2$

(C) $n = -1$ and $n = 1$

D $n = -3$ and $n = 1$

E $n = -2$ and $n = -1$

Figure 6-11. Algebra: Polynomials and Rational Expression items.

Algebra: Equations and Inequalities

The third, and largest, collection of algebraic problems was generally well-defined. The results on three items require elaboration. Although Form 7, item 10 had a very high non-response rate, well over half of the students who attempted it were successful. No great knowledge of algebra is required for this item; all that is necessary is to enumerate the six different combinations of two natural numbers summing to four or less. It is apparent that many students were diverted by the apparent algebraic complexity of having to deal with three inequalities.

On Form 8, item 13, students may have failed to recognize that the correct answer $(x - x_1)(x - x_2) < 0$, is equivalent to the statement $x_1 < x < x_2$. Those students who felt that the latter form was correct may have chosen option E, "none of these", this option being the most popular of the five.

Another item of interest is Form 2, item 8. This item had the lowest OTL of the group, yet students did reasonably well. The item has the appearance of an applied problem requiring the solution of a quadratic inequality, yet the solution, as in the case of Form 7, item 10, may be arrived at by constructing a table of values of six ordered pairs.

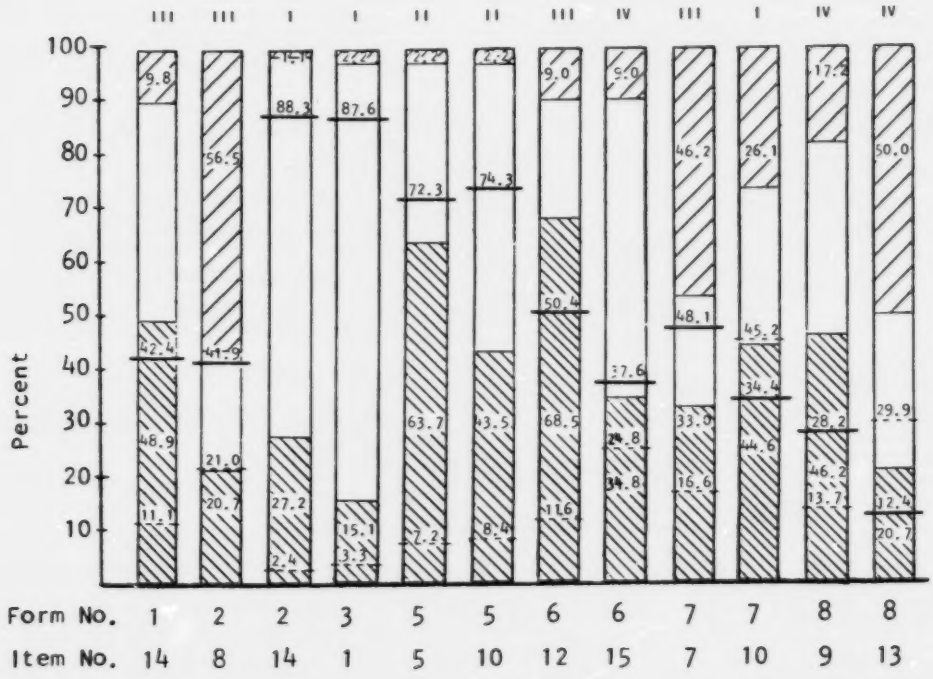


Figure 6-12. Results on Algebra: Equations and Inequalities.

1-14. Which of the following points lies in the region bounded by the lines $y = 1$, $y = x$, and $x + y = 6$.

(A) (3,2)

B (4,3)

C (1,5)

D (3,4)

E (2,3)

2-8. Two mathematical models are proposed to predict the return y, in dollars, from the sale of x thousand units of an article (where $0 < x < 5$). Each of these models A and B, is based on different marketing methods.

model A $y = 6x - x^2$
model B $y = 2x$

For what values of x does model B predict a greater return than model A?

A $0 < x < 4$

B $0 < x < 5$

C $3 < x < 5$

D $3 < x < 4$

(E) $4 < x < 5$

2-14. The solution set for the equation $(1 - 2x)(2 + x) = 0$ is

(A) $(\frac{1}{2}, -2)$

B $(-\frac{1}{2}, 2)$

C $(-1, -2)$

D $(\frac{1}{2}, 2)$

E $(2, -2)$

3-1. For the equation $x^2 - 5x + 6 = 0$

A there is no solution

B there is exactly one solution

(C) there are exactly two solutions

D there are exactly three solutions

E there are more than three solutions

5-5. What are all values of x for which the inequality

$5x + \frac{1}{3} \leq -2x - \frac{2}{3}$

is true?

A $x \leq -\frac{7}{8}$

(B) $x \leq -\frac{1}{3}$

C $x \geq 0$

D $x \geq \frac{7}{3}$

E $x \geq \frac{8}{3}$

5-10.

The equation of line ℓ is $y = 4x - 5$.
The equation of line m is $y = 2x + 2$.
What is the solution of the simultaneous equations

$y = 4x - 5$
$y = 2x + 2$

(A) The coordinates of P_1

B The coordinates of P_2

C The coordinates of P_3

D The x-value at P_2 and the y-value at P_3

E The y-value at P_2 and the x-value at P_3

6-12. A freight train travelling at 50 kilometres per hour leaves a station 3 hours before an express train which travels in the same direction at 90 kilometres an hour. How many hours will it take the express train to overtake the freight train?

A $\frac{5}{3}$

B $\frac{9}{5}$

C $\frac{12}{5}$

(D) $\frac{15}{4}$

E $\frac{18}{4}$

6-15. A piece of wire 52 cm long is cut into two parts and each part is bent to form a square. The total area of the two squares is 97 cm^2. How much longer (in cm) is a side of the larger square than a side of the smaller square?

A 4

(B) 5

C 9

D 14

E 20

7-7. A certain number of students are to be accommodated in a hostel. If two students share each room, then two students will be left without any room. If three students share each room, then two rooms will be left unoccupied. How many rooms are there in the hostel?

A 6

(B) 8

C 10

D 12

E 14

7-10. The number of pairs of integer values of x, y which satisfy $x + y \leq 4$, $x > 0$, $y > 0$ simultaneously is

A 3

(D) 6

C 10

D 14

E 15

8-9. A stationer wants to make a card B cm long and of such a width that when the card is cut into halves the original width becomes the length and the shape of each half is similar to the original card. What width, in centimetres, should he make the original card?

A 4

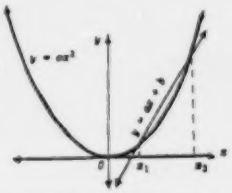
(B) $4\sqrt{2}$

C $5\sqrt{2}$

D $5\sqrt{3}$

E 6

8-13.



According to the graph, $ax + b > ax^2$ when

A $(x - x_1)(x - x_2) > 0$

(B) $(x - x_1)(x - x_2) < 0$

C $0 < x < x_1$

D $x > x_2$

E None of these

Figure 6-13. Algebra: Equations and Inequalities items.

Functions

This collection of items demonstrated a wide range of OTL, p-values, and non-response rates. With two exceptions, achievement levels were relatively low.

For Form 1, item 15 teachers indicated a low OTL, yet the definition contained all the information necessary to answer the question, provided the student had a basic understanding of functions. Although almost half of the students avoided the question, over half of those who tried it were successful. This type of question is similar to Form 2, item 16 shown in Figure 6-6, in which a definition was given and students were asked to reason from it rather than to recall some previous knowledge which could be applied to the problem.

Similar results occurred on Form 3, item 13, but most teachers taught the material during the year. This was a routine exercise in the application of exponential functions. The avoidance tactic of students likely is a reflection of the fact that the basic textbook contains little work that deals with the applications of exponential and logarithmic functions. The textbook approach is to pose "problems" in order to increase computational skills using tables of logarithms, a practice that is increasingly questionable in the age of scientific calculators.

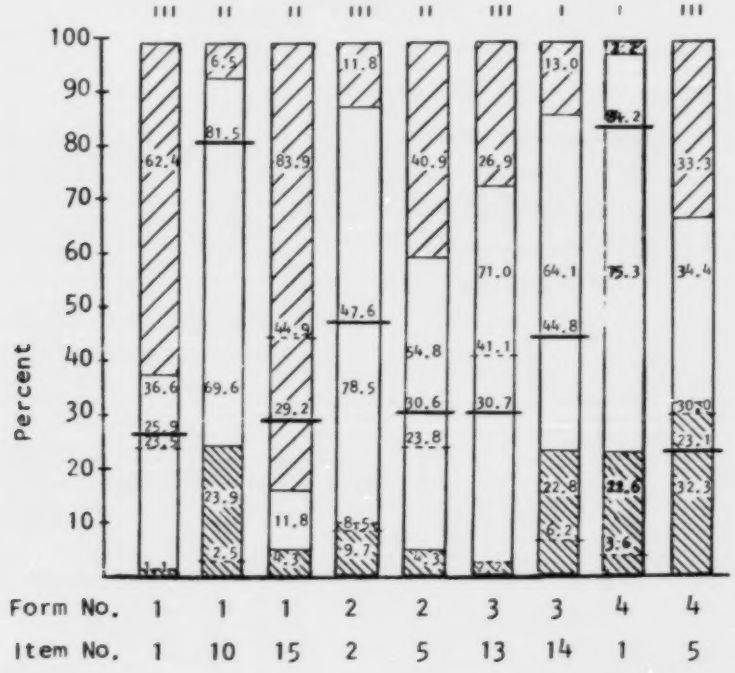


Figure 6-14. Results for Functions (Form 1, item 1, to Form 4, item 5).

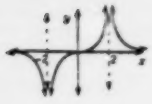
1-1. Which of these is the sketch of the graph of the function f where

$$f(x) = \frac{x^2}{(x-2)(x+2)}$$

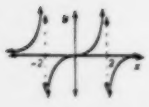
A

B

C



D



E

1-10. If $a_0 = 1$ and a is greater than 0, which of the following statements is true?

A When x is greater than 1, y is negative.

B When x is greater than 1, y is greater than 1.

C When x is less than 1, y is less than 1.

D As x increases, y increases.

(E) As x increases y decreases.

1-15. A real valued function f defined on a set of real numbers is said to be additive if, for all x and y in the domain of f,

$$f(x + y) = f(x) + f(y)$$

If the function f defined on the set of positive real numbers is

A $f(x)=x^2$, then f is additive.

B $f(x)=\sin x$, then f is additive.

C $f(x)=\log_{10} x$, then f is additive.

D $f(x)=2^x$, then f is additive

(E) None of the above is additive

2-2. The curve defined by $y = 3x(x - 2)/(2x + 1)$ intersects the x-axis only at the points

A $(-2,0)$ and $(\frac{1}{2}, 0)$

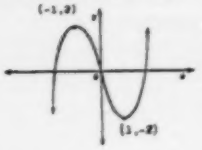
B $(2,0)$ and $(-\frac{1}{2},0)$

C $(3,0)$ and $(-2,0)$ and $(\frac{1}{2},0)$

D $(3,0)$ and $(2,0)$ and $(-\frac{1}{2},0)$

(E) $(0,0)$ and $(2,0)$ and $(-\frac{1}{2},0)$

2-5. The diagram shows the sketch of the graph of the cubic function f.



The function f could only be given by

A $f(x) = -x^3 - x$

B $f(x) = x^3 - 3x^2$

(C) $f(x) = x^3 - 3x$

D $f(x) = 3x^3 - x$

E $f(x) = x^3 + 3x^2$

3-13. A radio-active element decomposes according to the formula

$$y = y_0 e^{-kx}$$

where y is the mass of the element remaining after x days and y_0 is the value of y for $x = 0$. Find the value of the constant k for an element whose half-life (i.e. time to decompose half of the material) is 4 days.

(A) $\frac{1}{4}\log_e 2$

B $\log_e \frac{1}{2}$

C $\log_2 e$

D $(\log_e 2)^{\frac{1}{4}}$

E $2e^4$

3-14. The inverse of the function $\{(-2,0), (-1,1), (0,2), (1,3)\}$ is

A $\{(2,0), (1,-1), (0,-2), (-1,-3)\}$

(B) $\{(0,-2), (1,-1), (2,0), (3,1)\}$

C $\{(\frac{1}{2},0), (-1,1), (0,\frac{1}{2}), (1,\frac{1}{3})\}$

D $\{(1,3), (0,2), (-1,1), (-2,0)\}$

E $\{(0,2), (1,1), (2,0), (-3,-1)\}$

4-1. A function f with domain $\{1,2,3\}$ is defined by $f(x) = 2^x$. The range of f is

A $\{\frac{1}{4}, \frac{1}{2}, \frac{1}{8}\}$

B $\{\frac{1}{2}, 1, 1\frac{1}{2}\}$

C $\{1, 2, 4\}$

D $\{2, 4, 6\}$

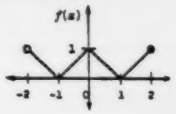
(E) $\{2, 4, 8\}$

4-5. A function f is defined by

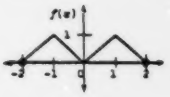
$$f(x) = \begin{cases} -x - 1 & \text{if } -2 \le x \le -1 \\ x + 1 & \text{if } -1 < x \le 0 \\ -x + 1 & \text{if } 0 < x \le 1 \\ x - 1 & \text{if } 1 < x \le 2 \end{cases}$$

which is the graph of f

(A)



B



C

D

E

Figure 6-15. Functions items (Form 1, item 1, to Form 4, item 5).

Form 4, item 5 was a difficult item, having a high non-response rate. Both of these characteristics may be due to the sheer tedium of working one's way systematically through the various possibilities.

On Form 8, item 17, the percent of students correctly answering this item was lower than the percent who did not respond to it. The low p-value likely is due to the presence of a highly attractive distractor, (B), making the item the most difficult of the entire set.

On the whole, achievement on this topic was low, with only a few exceptions, and this is rather disappointing. One should expect, for example, much better performance on items such as Form 4, item 10; Form 7, item 12; and Form 8, item 12. All require only elementary knowledge of graphs and functions.

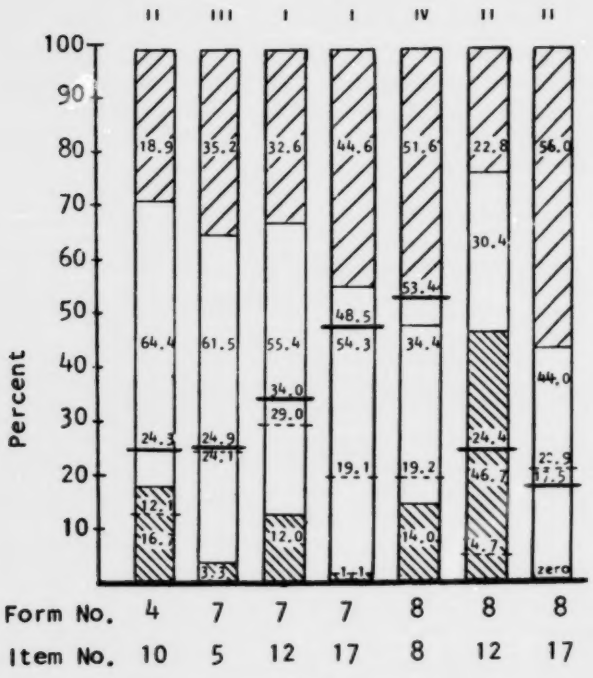
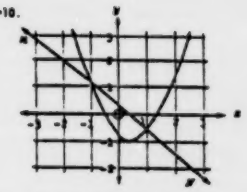


Figure 6-14 (cont'd). Results on Functions (Form 4, item 10 to Form 8, item 17).

4-10.



For what values of x does the function represented by the straight line $\overline{AB}$ exceed the quadratic function?

(A) $-1 < x < 1$

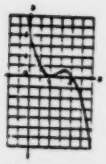
B $x < -1$ or $x > 1$

C $-\frac{3}{4} < x < 1\frac{1}{4}$

D $x > 0$

E $x > y$

7-5.



The graph above is the representation of one of the following equations. Which one does it represent?

A $y = (1 - x)(x - 2)$

B $y = (1 - x)(2 - x)$

C $y = (1 - x)(2 - x)^2$

D $y = (1 - x)^2(x - 2)$

(E) $y = (1 - x)^2(2 - x)$

7-12. The graph of the curve $y = \frac{2x + 1}{x^2 + 2x + 3}$ intersects the axes at the points

A $(\frac{1}{2}, 0)$ and $(0, -\frac{1}{3})$

B $(-\frac{1}{2}, 0)$ and $(0, \frac{1}{3})$

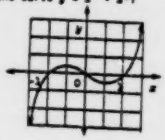
C $(\frac{1}{2}, 0)$ and $(0, \frac{1}{3})$

D $(-\frac{1}{2}, \frac{1}{3})$ and $(0, 0)$

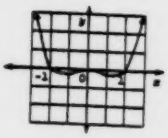
(E) $(-\frac{1}{2}, 0)$ and $(0, \frac{1}{3})$

7-17. Which of the following could be a sketch of the graph of the curve $y = x^3 - x^2$?

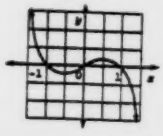
A



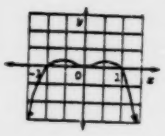
(B)



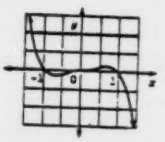
C



D



E



8-8. Let a function f be defined by $f(x,y) = x$.
Let $G = \{(x,y) \mid f(x,y) = 2\}$.
The graph of G is

(A) the line $x = 2$

B the line $x + y = 2$

C the line $x = y$

D the x-axis

E the y-axis

8-12. The functions f and g are defined by $f(x) = x - 1$ and $g(x) = (x + 3)^2$

$g(f(x))$ is equal to

A $(x - 1)(x + 3)^2$

B $(x + 3)^2 - 1$

C $(2x - 2)^2$

(D) $(x + 2)^2$

E $x^2 + 8$

8-17. Let $y = ax^3$, with x and y taking positive real values. When $\log x$ is plotted against $\log y$, the resulting graph will be

A a single point

B a cubic curve

C a parabola

(D) a straight line

E an exponential curve

Figure 6-15 (cont'd). Functions items (Form 4, item 10, to Form 8, item 17).

Analytic Geometry and Vectors

Relatively low p-values were indicated on three items in this group of ten. On six of the remaining items students consistently scored between 50% and 60% correct.

Two items are marked by low OTL, coupled with reasonably good achievement. Success on Form 6, item 7 could have been achieved through an intuitive feeling for a rotation in space rather than through formal instruction. Form 8, item 11 could have been answered correctly by intelligent guessing, without a knowledge of ordered pair vector notation.

A high non-response and a low p-value were obtained on Form 7, item 13. The item looks formidable in its abstraction, but it requires only a careful application of a basic knowledge of the coordinate plane. There may have been some negative carry-over to the following item, a similar type of problem, which many students avoided.

A major disappointment is performance on Form 7, item 2, which requires the calculation of the slope of a line given two points on it. The most popular incorrect distractor was the reciprocal of the correct answer, perhaps indicating just carelessness in selecting variables. But this should not be occurring with a select group of Grade 12 students responding to a Grade 10 level item.

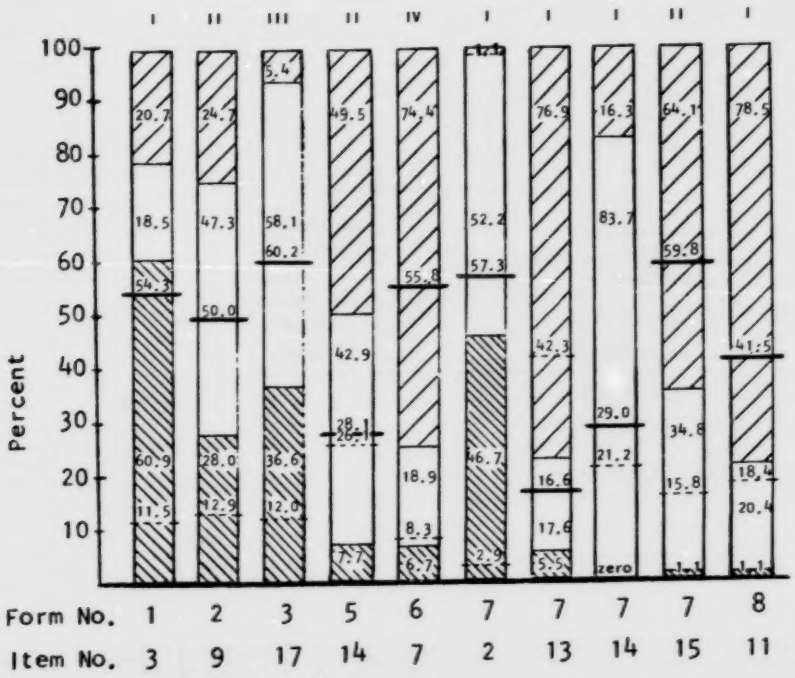


Figure 6-16. Results for Analytic Geometry and Vectors.

1-5.

If $UV = 2$, $YZ = 3$, $XU = 3$ and $UV \parallel YZ$, then UY is equal to

A $4\frac{1}{2}$

B 3

C 2

(D) $1\frac{1}{2}$

E $\frac{2}{3}$

2-9. One side of an equilateral triangle lies along the x-axis. The sum of the slopes of the three sides is

(A) 0

B -1

C 1

D $2\sqrt{3}$

E $1 + 2\sqrt{3}$

3-17. In a Cartesian coordinate system, what is the equation of the straight line passing through the point $(0,-5)$ and parallel to the straight line whose equation is $y = 2x + 3$?

A $x + 2y + 5 = 0$

(B) $2x - y - 5 = 0$

C $2x + 3 = -5$

D $2x - \frac{1}{2}y + 3 = 0$

E $2x + y + 5 = 0$

5-14. The rectangular coordinates of three points in a plane are $Q(-3,-1)$, $R(-2,3)$ and $S(1,-3)$. A fourth point T is chosen so that $\overline{ST} = 2\overline{QR}$. The y-coordinate of T is

A -11

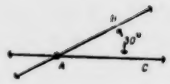
B -7

C -1

D 1

(E) 5

6-7.



As $\overline{AB}$ rotates about $\overline{AC}$ in space, keeping an angle of $30°$, what figure is traced out by the line $\overline{AB}$?

(A) a cone

B a cylinder

C a spiral

D a circle

E a sphere

7-2. The slope of the line through the two points $(-1,3)$ and $(4,-7)$ is

A $-\frac{1}{2}$

B $-\frac{3}{4}$

C $-\frac{4}{3}$

(D) -2

E $-\frac{10}{3}$

7-13. l is the line determined by the equation $ax + by = 0$, and m is the line determined by the equation $px + qy + r = 0$, where $r \neq 0$. If l and m intersect only at the point P, and O is the origin, then $(a + p/x + (b + q/y + r = 0$ is a line which

A is mutually perpendicular to l and m

B forms an isosceles triangle with l and m

C is parallel to $\overleftrightarrow{OP}$

D passes through the origin

(E) passes through P

7-14. For which of the following values of m is the graph of $y^2 - 2y + mx^2 + (2m + 1)x = 0$ a parabola?

A $m = -\frac{1}{2}$

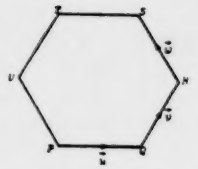
(B) $m = 0$

C all values of m except $-\frac{1}{2}$

D all values of m except 0

E all values of m except $-\frac{1}{2}$ and 0

7-15.



PUNSTU is a regular hexagon in which $\overrightarrow{RS}$ represents vector $\vec{u}$, $\overrightarrow{QR}$ represents vector $\vec{v}$, and $\overrightarrow{RS}$ represents vector $\vec{w}$. $\overrightarrow{PT}$ represents

A $2\vec{u} + \vec{v}$

B $\vec{u} + \vec{v} + \vec{w}$

C $2(\vec{u} + \vec{v} + \vec{w})$

D $2\vec{u} + \vec{v} + \vec{w}$

(E) $\vec{v} + \vec{w}$

8-11. Find the difference $\vec{b} - \vec{a}$ of the vectors $\vec{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

A $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

(B) $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

C $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$

D $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

E $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Figure 6-17. Analytic Geometry and Vectors items.

Trigonometry and Complex Numbers

Out of the 14 items in this section, four had low OTL rates, with one as low as 10%. On all of these items the non-response rate exceeded the percent correct. All four items dealt with a single topic, the representation and manipulation of complex numbers in the complex plane. It seems clear that student exposure to complex numbers is basically at the level of algebraic symbolism. On such items, for example, Form 3, item 2, students did well. The major textbook used makes little attempt to show, in the complex plane, the effects of addition or exponentiation. The present secondary school program might be improved through the integration of the topics of trigonometry, complex numbers, and vectors.

On Form 4, item 7, students had to apply two trigonometric identities, always a difficult procedure. On Form 8, item 14 the word "unique" may have given students trouble. On this item, distractor E was the most effective, probably because it sets out two conditions, one of which is true.

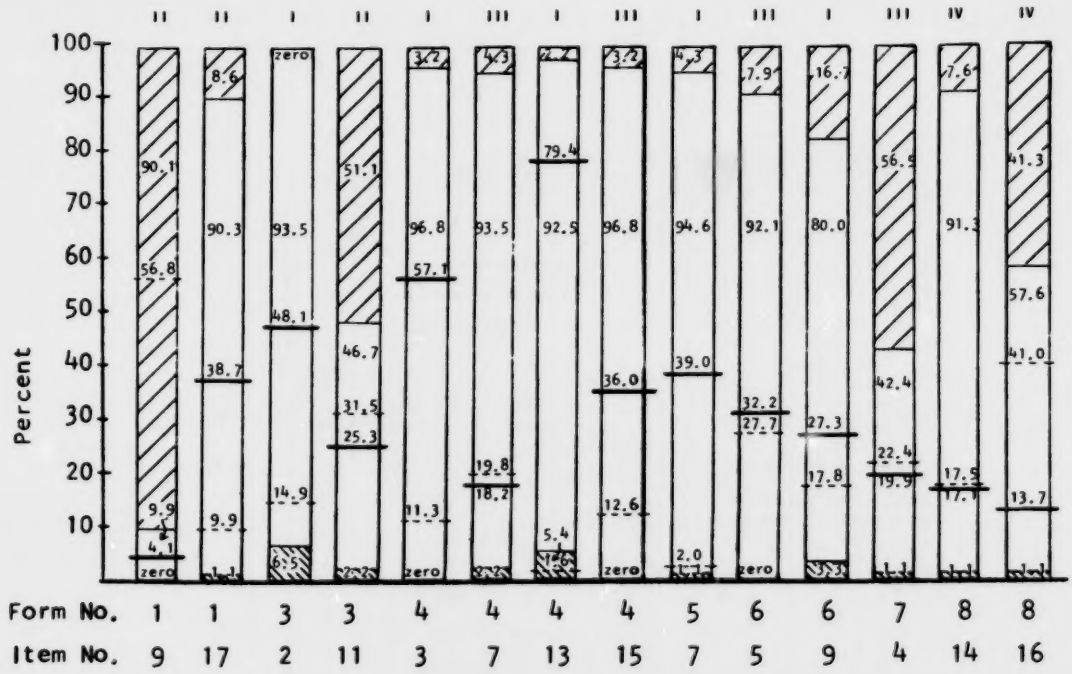


Figure 6-18. Results for Trigonometry and Complex Numbers.

1-9. a is a complex number with modulus $\sqrt{2}$ and argument $\frac{3\pi}{4}$. a is equal to

A $\frac{i-1}{\sqrt{2}}$

(B) $i-1$

C $\sqrt{2}(i-1)$

D $i+1$

E $\frac{i+1}{\sqrt{2}}$

1-17. Which of the following pairs of angles can be used as a COUNTEREXAMPLE to show that

$\sin(A+B) = \sin A + \sin B$ is not always true?

A $A = 30^\circ, B = -30^\circ$

B $A = 180^\circ, B = 180^\circ$

C $A = 90^\circ, B = 360^\circ$

D $A = 90^\circ, B = 270^\circ$

(E) $A = 90^\circ, B = 90^\circ$

3-2. If $a = 3+i$, and $b = -2-2i$, then $\frac{a}{b}$ is equal to

A $-1+4i$

(B) $-1+\frac{1}{2}i$

C $-1-\frac{1}{2}i$

D $-2+i$

E $-2-i$

3-11. If $a = \frac{\sqrt{3}+i}{2}$, then a^{48} is equal to

A i

B $-i$

C 0

D -1

(E) 1

4-3. Which of the following is negative?

A $\sin\frac{5\pi}{12}$

B $\sin\frac{5\pi}{8}$

(C) $\tan\frac{5\pi}{8}$

D $\tan\frac{5\pi}{4}$

E $\cos\frac{5\pi}{4}$

4-7. An angle θ is known to be between 90° and 180° and $\cos^2\theta = \frac{16}{25}$.

The value of $\sin 2\theta$ is then

(A) $-\frac{24}{25}$

B $-\frac{15}{25}$

C $-\frac{7}{25}$

D $\frac{7}{25}$

E $\frac{24}{25}$

4-13. The complex number $(1+i)^2$ is equal to

A 0

B 2

(C) $2i$

D $1+i$

E $2+2i$

4-15. Which of the following is (are) true for all values of θ for which the functions are defined?

I $\sin(-\theta) = -\sin\theta$
II $\cos(-\theta) = -\cos\theta$
III $\tan(-\theta) = -\tan\theta$

A I only

B II only

C III only

(D) I and III only

E II and III only

5-7. If $\cos\theta = \frac{1}{2}$, then $\cos 2\theta$ is equal to

(A) $-\frac{1}{2}$

B $\frac{1}{2}$

C $-\frac{\sqrt{3}}{2}$

D $\frac{\sqrt{3}}{2}$

E 1

6-5. In the interval $2\pi \le x \le 4\pi$, the solution set of the equation $\sin x = \frac{1}{2}$ is

A $(\frac{7\pi}{3}, \frac{8\pi}{3})$

B $(\frac{7\pi}{3}, \frac{11\pi}{3})$

(C) $(\frac{13\pi}{6}, \frac{17\pi}{6})$

D $(\frac{13\pi}{4}, \frac{19\pi}{6})$

E $(\frac{13\pi}{6}, \frac{23\pi}{6})$

6-9.

In the figure above, $\overline{PQ} \perp \overline{OQ}$ and $\overline{RS} \perp \overline{OQ}$. If the measures of $\overline{OQ}$ and of $\overline{QP}$ equal 1 and θ is the measure of $\angle POQ$, then the measure of segment $\overline{RS}$ is equal to

A $\sin\theta$

B $\cos\theta$

(C) $\tan\theta$

D $2\sin\theta$

E $1-\cos\theta$

7-4. If $a = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$, then a^3 is equal to

A 0

B 1

(C) i

D $\frac{\sqrt{3}}{8} + \frac{i}{8}$

E $\frac{\sqrt{3}}{8} - \frac{i}{8}$

8-14. In any triangle ABC, if a, b, and c are the lengths of the sides opposite angles A, B and C respectively, the following relationship is always true:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

If b, c and $\angle B$ are given, with $b < c$ and $\angle B$ acute, which of the following MUST be true to give unique solutions for the measures of $\angle A$ and $\angle C$?

(A) $b = c\sin B$

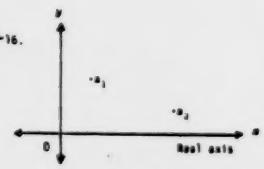
B $\frac{\sin A}{\sin B} < 1$

C $b \le c$

D $\sin B > \frac{b}{c}$

E $b \le c$ and A is an acute angle

8-16.



The complex number $a = x + iy$ (where x and y are real numbers) can be represented by the point (x,y). In the diagram, a_1 and a_2 represent two complex numbers. If a_3 is selected such that the origin, a_1, a_2 and a_3 are the consecutive vertices of a parallelogram, then a_3 is equal to

A $a_1 a_2$

B $\frac{a_1}{a_2}$

(C) $a_1 + a_2$

D $a_1 - a_2$

E $\sqrt{a_1^2 + a_2^2}$

Figure 6-19. Trigonometry and Complex Numbers items.

Permutations, Combinations, Probability, and Statistics

The results for this collection of nine items show that a miscalculation was made in considering these to be B. C. curriculum items. For only two, or possibly three, of the items is there sufficient evidence from OTL, percent correct, and percent non-response rates to suggest that the items are satisfactorily dealt with in the mathematics program.

Form 1, item 6, Form 5, item 9, Form 7, item 3, Form 8, item 10, all deal with permutations and combinations. The answer to Form 5, item 9 can be deduced by the simple expedient of trying a single case and matching results to the distractors. On the other three items, techniques of combinatorics must be used. The results show that students have not been exposed to these techniques. This is an optional topic on the B.C. syllabus in a crowded curriculum, so it is not surprising that this area has not been covered.

Form 1, item 8 and Form 6, item 10 are simple problems in descriptive statistics not requiring any specific knowledge apart from the definition of "average". The remaining three items dealt with the topic of probability. Form 2, item 12 and Form 4, item 14 were solvable using what might be termed "intuitive" notions of probability, whereas Form 4, item 11, which many students avoided, required a formal understanding of probabilistic reasoning. On the basis of this analysis, it seems unlikely that probability is being taught to any extent in the Grade 12 program.

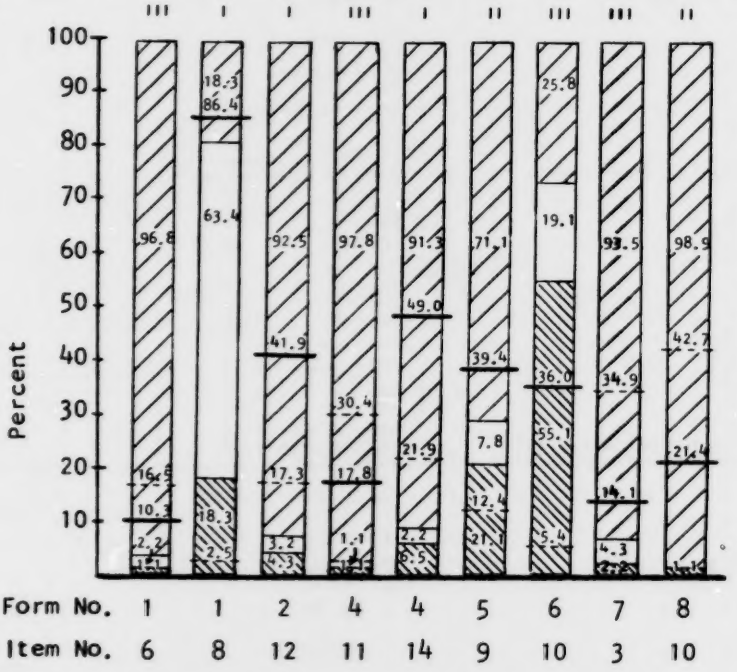


Figure 6-20. Results for Permutations, Combinations, Probability, and Statistics.

1-6. In how many ways can one arrange on a bookshelf 5 thick books, 4 medium sized books and 3 thin books so that the books of the same size remain together?

(A) $5! \times 4! \times 3! \times 3! = 103680$

B $5! \times 4! \times 3! = 17280$

C $(5! \times 4! \times 3!) \times 3 = 51840$

D $5 \times 4 \times 3 \times 3 = 180$

E $2^{12} \times 3 = 12288$

1-8.

In the graph, rainfall (in centimetres) is plotted for 13 weeks. The average weekly rainfall during the period is approximately

A 1 centimetre

(B) 2 centimetres

C 3 centimetres

D 4 centimetres

E 5 centimetres

2-12. Four persons whose names begin with different letters are placed in a row, side by side. What is the probability that they will be placed in alphabetical order from left to right?

A $\frac{1}{120}$

(B) $\frac{1}{24}$

C $\frac{1}{12}$

D $\frac{1}{6}$

E $\frac{1}{4}$

4-11. A warning system installation consists of two independent alarms having probabilities of operating in an emergency of 0.95 and 0.90 respectively. Find the probability that at least one alarm operates in an emergency.

(A) 0.995

B 0.975

C 0.95

D 0.90

E 0.855

4-14. A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?

A $\frac{1}{6}$

B $\frac{5}{24}$

C $\frac{1}{4}$

(D) $\frac{1}{3}$

E $\frac{5}{12}$

5-9. If n distinct points are marked on a circle, how many chords may be obtained by joining them?

A n

B $\frac{n}{2}$

C $n - 1$

D $n(n - 1)$

(E) $\frac{n(n-1)}{2}$

6-10. The same test was given in two classes. The first class, with 20 pupils, obtained an average score of 12.3 points. The second class, which had 30 pupils, obtained an average score of 14.8 points. What was the average score of the whole group of 50 pupils?

A 12.55

B 13.3

C 13.55

(D) 13.8

E 14.3

7-3. How many four-digit numbers less than 2467 can be formed from the digits 1, 2, 3, 4 using each digit once?

(A) $2 \times 3 \times 2 \times 1$

B $2 + 3 + 2 + 1$

C $4 \times 3 \times 2 \times 1$

D $4 + 3 + 2 + 1$

E $1 \times 3 \times 2 \times 1$

8-10. An examination consists of 13 questions. A student must answer only one of the first two questions and only nine of the remaining ones. How many choices of questions does he have?

A ${}_{13}C_{10} = 286$

B ${}_{11}C_{9} = 165$

(C) $2 \times {}_{11}C_{9} = 110$

D $2 \times {}_{11}P_{9} = 220$

E some other number

Figure 6-21. Permutations, Combinations, Probability and Statistics items.

6.3 Achievement by Cognitive Level

In the design of the IEA instruments great care was taken to ensure that questions covered a range of cognitive levels. The four levels for which items were written were: I. Computation, II. Comprehension, III. Application, and IV. Analysis. Information related to these levels is contained in Table 6-3.

Table 6-3
Achievement Results by Cognitive Level

Cognitive Level		Number of Items	Mean Percent Correct	Mean Percent Non-response	Mean Percent Never Taught
I.	Computation	31	51.3	13.3	22.5
II.	Comprehension	20	44.3	13.1	24.4
III.	Application & Analysis	32	37.0	18.0	26.3
	TOTAL	83	44.1	15.1	24.4

Categories three and four have been grouped together due to the small number of items in the fourth category, and the items which were identified in the previous section as not taught in B. C. schools have been eliminated. There is a consistent trend toward increasing difficulty with increasing cognitive complexity. Students appear more reluctant to attempt the higher cognitive level items. Teachers tend to suggest that material related to higher cognitive level items was not taught. This latter finding is similar to that found for the Grade 8 classes, although differences are smaller at the Grade 12 level. This may indicate that the Grade 12 teachers had a better understanding of what was required in assessing opportuntiy to learn.

CHAPTER 7

STUDENT ATTITUDES: GRADES 8 AND 12

In addition to an evaluation of achievement, the Second International Mathematics Study included a survey of student attitudes toward a number of issues, ranging from specifics of the mathematics curriculum to the role of mathematics in our society. Since most of the attitude scales used were identical for the two grade levels, it is possible to compare the results across the groups. Furthermore, a limited teacher-student comparison is possible, since teachers also responded to several of the attitude items.

7.1 Description of the Instruments

Seven different sets of actitude questions were designed around the following topics:

Mathematics in School
Mathematics as a Process
Parental Support for Mathematics
Mathematics and Myself
Mathematics and Society
Mathematics and Gender
Computers and Calculators

Items for the SIMS were selected from the First IEA mathematics study, from the National Assessment of Educational Progress in the U.S.A., and from existing mathematics attitude scales. Try-outs were first conducted in the U.S.A., and these were followed by piloting on an international scale. The analyses indicated satisfactory results for reliability, but it was felt there was little direct evidence that students from different countries responded with the same underlying feeling for what the thing we call "mathematics" really is. For the B. C. report, in which no international comparisons are made, one must rely solely on the face validity and statistical reliability for assurances that the measures really do mean something.

7.2 Mathematics in School

The items in this group were designed to measure attitudes toward specific mathematical topics and activities, on three different dimensions. As may be seen in Figure 7-1, for each activity, students were asked to indicate how they felt with respect to the importance of the activity, its difficulty, and their liking for it.

Checking an answer to a problem by going back over it.

(a)	very important	important	undecided	not important	not at all important
(b)	very easy	easy	undecided	hard	very hard
(c)	like a lot	like	undecided	dislike	dislike a lot

Figure 7-1. Example of a Mathematics in School item.

Grade 8 results

Students' attitudes toward fifteen mathematical activities were assessed at the Grade 8 level. These were:

*1. Checking an answer to a problem by going back over it.
*2. Memorizing rules and formulas.
3. Using charts and graphs.
*4. Solving word problems.
5. Solving equations.
6. Solving inequalities.
7. Learning about geometric figures.
*8. Estimating answers to problems.
9. Learning about ratio and proportion.
10. Working problems that involve decimal fractions.
11. Working with sets.
12. Learning about units of measurement (e.g., distance, area, volume)
13. Drawing geometric figures.
14. Getting information from statistical tables.
15. Comparing geometric figures that are similar.

For those items marked with asterisks, teachers were asked to indicate how they felt about teaching each of these mathematical activities.

In Figure 7-2, the student and teacher posttest results at the end of the school year have been plotted in a three dimensional grid.[1] The three dimensions--importance, ease,

[1]The authors are indebted to Les MacLean of OISE for suggesting this form for presenting the data.

and liking--have been scaled so that the five options for each item take values from -2 to 2, with the positive values indicating more importance, easier, and greater liking for each item.

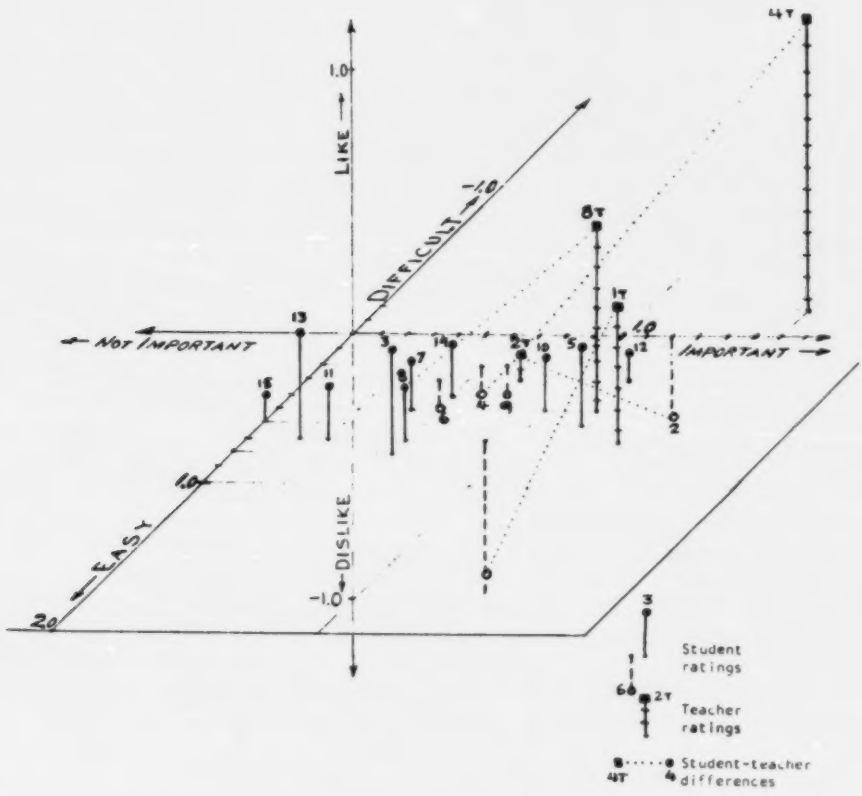


Figure 7-2. Grade 8 item ratings for Mathematics in School.

The horizontal plane suggests a neutral position of like or dislike toward an activity and hence might be termed the "plane of indifference". Activities which students tended to like are located above this plane and those which they disliked are below. The distance above, indicated by solid lines, or below, indicated by dashed lines, indeed all distances, represent the extent to which opinions differed from neutrality. No clear structure emerges, but several generalizations may be hazarded. No activity was rated as unimportant, nor was any rated as difficult. In fact, all of the items are in the range from 0 to 1 in easiness. The range of like/dislike is also about one unit, but split above and below the plane of indifference.

Activities 1 and 2 were the most disliked, and they might be characterized as mechanical processes. Activity 4 as well, solving word problems, was not liked. The responses teachers made to these three items is instructive. On item 1 teachers and students agreed on the easiness of teaching and learning how to check answers but they disagreed on the level of importance, and more distinctly, on their liking for the process. For item 4 the difference on the same two dimensions was even more marked, particularly on the like/dislike scale. Still there, but to a lesser extent, was disagreement on item 2, memorizing rules and formulas.

Students responded to this particular set of items in October, 1980 and again in May, 1981. As a result it is possible to determine global changes in attitudes over the academic year. In general, there was little change in attitude. On just four items were differences of more than 0.1 on any scale indicated from pretest to posttest. For item 1 students showed an increased dislike, with 58% disliking it at the end of the year compared to 45% at the beginning. This was not accompanied by any major shift on the other two scales. Students also showed, but to a lesser extent, the same tendency on item 5. On item 6, solving inequalities, there was a tendency to more importance coupled with an increased perceived difficulty, and dislike. On item 7, learning about geometric figures, students felt it became less significant, but easier.

Some correlational data are available from the pretest results, and are shown in Table 7-1.

Table 7-1
Correlations* Within 18 Items of "Mathematics and School"

Correlations Between:	Range Low	High	Median
Important/Easy	-0.02	0.28	0.14
Important/Like	0.25	0.38	0.36
Easy/Like	0.18	0.55	0.51

* Pearson's r, 5 values on each scale; n=2392

In general, there is a positive relationship among responses on the three scales, although the correlations between importance and ease are very small. The reasonably strong correlation between easiness and degree of liking should not be construed in any causal fashion, but the results add credence to the notion that students feel more confident with material that they like, or vice versa.

A second question was investigated through the correlational data, namely, to what extent did a student tend to respond in similar fashion to each item within a scale? That is, if a student perceived an item as important did he or she also tend to perceive the other items as important? For this, and the equivalent question on the easiness and liking scales, the correlations were about 0.24. This indicates that there is some tendency for students to perceive items as a whole from a particular point of view. The relationship, however, is not strong.

Grade 12 results

Students at the Grade 12 level were asked to respond to a set of items similar to the Grade 8 Mathematics in School collection. These items dealt with:

1. Checking an answer to a problem by going back over it.
2. Memorizing rules and formulas.
3. Solving word problems.
4. Getting information from statistical tables.
5. Solving equations.
6. Proving theorems.
7. Using vectors.
8. Working with complex numbers.
9. Investigating sequences and series.
10. Differentiating functions.
11. Drawing graphs of functions.
12. Finding a limit of a function.
13. Integrating functions.
14. Determining the probability of an outcome.
15. Using a hand held calculator.

Items 1 to 5 were also contained in the Grade 8 questionnaire. Teachers were asked to indicate how they felt about teaching the topics dealt with in items 1, 2, and 3. A diagram similar to that set out for Grade 8 is contained in Figure 7-3.

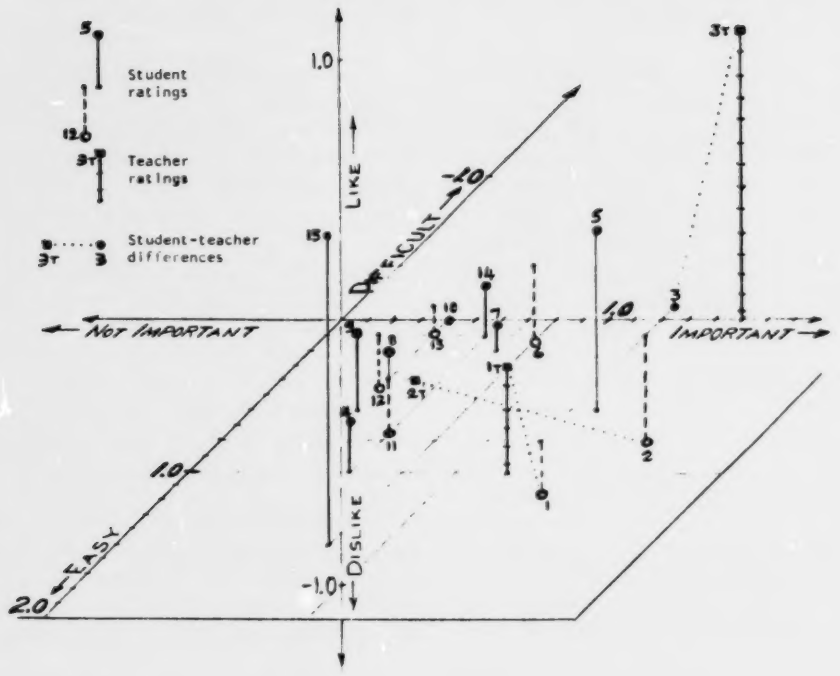


Figure 7-3. Grade 12 item ratings for Mathematics in School.

Most points have been classified by their location in one of a number of identifiable regions in space. These are:

1) The sphere of neutrality. This region contains those items lying on or within a sphere of radius 0.5 units from the origin. On these items students were basically undecided in rating them on the three dimensions. Items in this region are 10, 12, and 13. All three items are non-curriculum items for B. C.: differentiation, limits, and integration. All three showed a high proportion of missing data as well as a high proportion of undecided students.

2) The cube of comfort. This is the region defined by a cube of dimension one unit in the positive direction along each axis. Items lying on or within this space were rated as somewhat important, rather more easy than difficult, and liked rather than disliked. It is expected that students feel comfortable with such items, these being items 4, 7, 8, 9, and 14. The latter four form a close cluster measuring attitudes toward vectors, complex numbers, series, and probability. Although students rated item 4 as easy, their degree of experience with statistical tables may be limited. Grade 8 students also placed it in the same region.

3) The plane of indifference. This is the horizontal plane, which contains item 3, solving word problems. Students were neutral in attitude but tended to see word problems as difficult. In comparison with the Grade 8 students, students in Grade 12 viewed solving word problems as more important, more difficult, and better liked.

4) The space of dislike. This region includes all points lying below the plane of indifference. Items located here are 1, 2, 6, and 11. Items 1 and 2, checking answers, and memorizing rules, were similarly located at the Grade 8 level. Item 6, proving theorems, was perceived as the most difficult of all activities as well as relatively unimportant. It is a matter of curiosity as to the students' reference system for this judgement, since there is little left in the secondary school mathematics curriculum which involves such an activity.

A disappointing result was that on item 11, drawing graphs of functions. Such an activity is central to an understanding of relationships and lays the groundwork for future mathematics courses. Students disliked the activity and saw it as relatively unimportant. It is difficult to understand how a topic such as this, with potential for student activity and involvement, should be disliked to a degree second only to memorizing formulas.

5) Outliers. These are items which exhibit unique characteristics. In the Grade 12 space there are two, items 5 and 15.

Item 5, solving equations, was rated highest in importance as well as second highest in liking. In comparison to Grade 8, Grade 12 students rated it similarly on the importance and ease dimensions but showed a considerably greater degree of affection for the topic. This may be a result of increased familiarity and confidence with the mechanics of solving equations since the topic is introduced only at Grade 8.

The other outlier is item 15, using a hand held calculator. This activity was highly rated on all dimensions. The information provided here is corroborated by the Grade 12 results on the Calculators and Computers scale (see the latter part of this chapter). It cannot be stressed too highly that this is a fertile area for curriculum development. Many of the mathematical topics in the senior high school are admirably suited to the use of calculators for concept development, skill maintenance, and problem solving.

Student versus teacher perceptions

There were three items in common responded to by all students and teachers. The location of each point as rated by each group is shown in Figure 7-4.

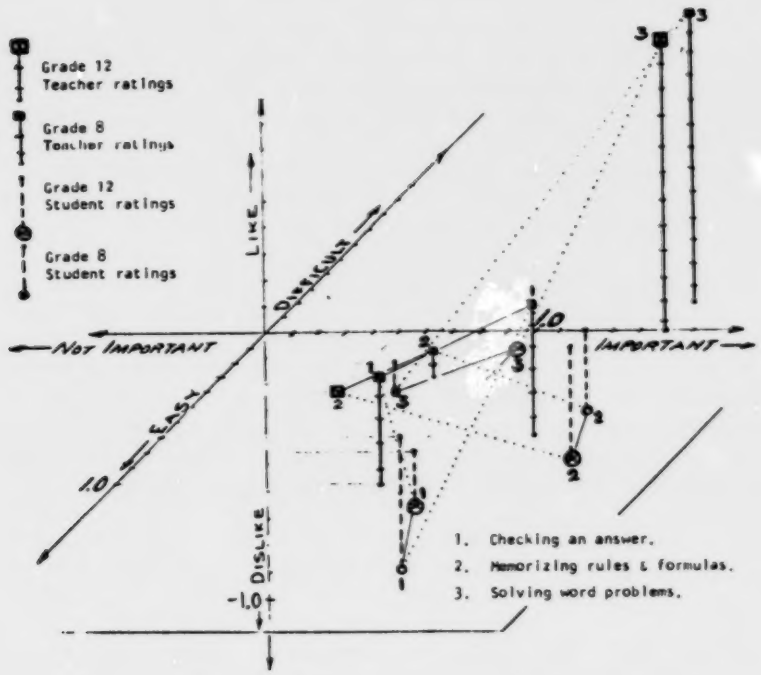


Figure 7-4. Comparison of Grades 8 and 12 student and teacher ratings of common items on Mathematics in School.

One may get a feeling for the degree of congruence between the teachers at the two levels and between students at different levels by examining the lengths of the solid connecting lines, with due regard for spatial effects. This may be compared to the length of the dashed lines, showing the separation of teachers and students within each level. In all cases the teacher-student separation is greater than that for teacher-teacher or student-student. All teacher estimates are on or above the plane, all student estimates are on or below.

The dichotomy of content versus process is evident in Figure 7-4. Teachers tend to be neutral about the need for memorizing rules and formulas, while students see it as important and dislike it.

Particularly serious is the mismatch on solving problems, a topic of major concern for the curriculum of the 1980s. Teachers and students are agreed that the teaching/learning of the topic is not easy, but there is clear disagreement as to its importance and the degree of liking evidenced for the process.

7.3 Mathematics as a Process

The items under this heading make up a single scale designed to measure how both students and teachers view the nature of mathematics as a discipline. These items were responded to by all the Grade 12 students and their teachers and also by the Grade 8 students and their teachers. All the results are reported in this section.

Respondents were asked the extent to which they agreed or disagreed with statements that characterize the field of mathematics in a variety of ways. Because some items were negatively worded, responses were re-coded on a 5-point scale as follows: 1. Strongly negative, 2. Negative, 3. Undecided, 4. Positive, 5. Strongly positive. On this scale, to be positive is to view mathematics as a field where speculation and heuristics are important rather than just rules; where the discipline is changing rather than fixed; and where there is opportunity for creative endeavor by imaginative persons. The results obtained from the four groups of respondents are shown in Table 7-2.

Students at both levels were similar in outlook; as were teachers at both levels. Teachers were more positive than students, and there was a trend toward increasingly positive attitude with, perhaps, experience with mathematics as the underlying variable. These results corroborate the pilot testing results obtained from eight different countries.

Table 7-2
Mathematics as a Process (15 items)

Group	Grade	Number	Mean	S.D.	Reliability*
Students	8	2173	3.05	0.51	0.78
Students	12	1956	3.13	0.46	0.72
Teachers	8	89	3.54	0.55	0.84
Teachers	12	97	3.64	0.33	0.54

* Hoyt's coefficient

Item results are presented in Figure 7-5. In this figure, and in others which follow, items marked with an asterisk are negatively worded. The items functioned reasonably well as a scale for all but the Grade 12 teachers. For that group, items 14 and 15 were particularly troublesome, with the teachers responding differently on those two items than on the remaining 13 items.

In general, teachers were more positive than students, and, within groups, those at the Grade 12 level viewed mathematics as a process slightly more favorably than those at the Grade 8 level. The overall scale trend was replicated on most items. The notable exception was item 1, Mathematics will change rapidly in the near future, in which the trend is reversed. One could speculate that respondents interpreted mathematics to be "school mathematics". It may be that students looked towards new mathematics topics while teachers were concerned with the process of curriculum change in the schools.

The largest systematic difference occurred on item 8: Learning mathematics involves mostly memorizing. Grade 8 students tended to agree; Grade 12 students disagreed, to an extent not seen on any other item. This tendency was also shown on item 11: There is always a rule to follow in solving a mathematics problem. As a consequence, it seems fair to say that students are less process-oriented than teachers, and within these groups, those at lower grade levels are less process oriented than those at the upper grade level. It is difficult, however, to characterize this change. It may be a result of aging, or experience with mathematics, or it may be a result of the selection process in which views towards mathematics may be a factor.

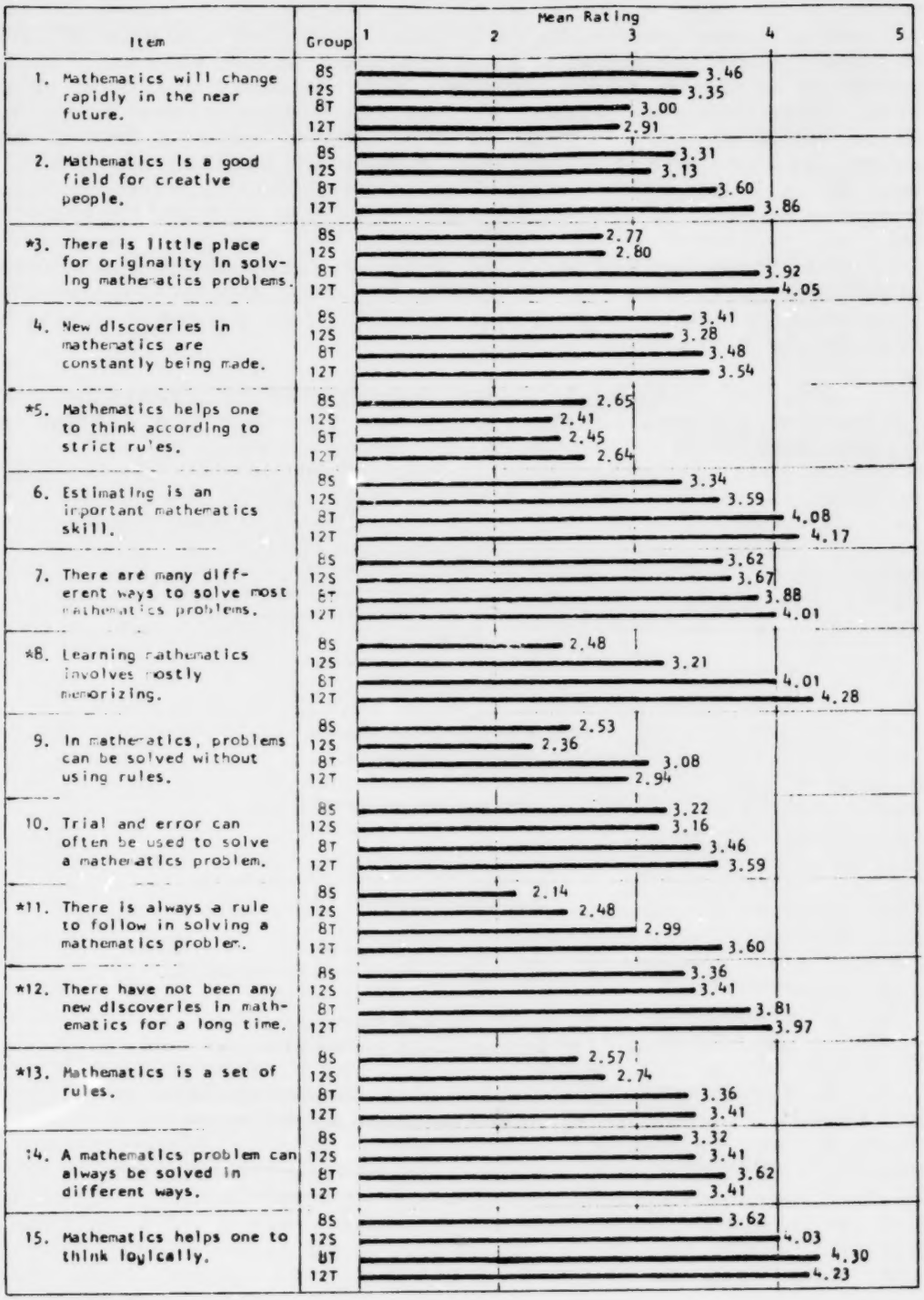


Figure 7-5. Item results for Mathematics as a Process.

7.4 Parental Support for Mathematics

Nine items were used to measure parental support of the student of mathematics. Students were asked to rate, on a scale of 1 to 5, how closely their parents matched each item. For example, item 1 stated "My father seems to enjoy doing mathematics". Each student indicated the degree to which this described his or her father by marking one of 1. Not at all like, 2. Not very much like, 3. Unsure, 4. Somewhat like, 5. Exactly like.

The items worked well as a scale, at both grade levels with Hoyt reliabilities of 0.79 and 0.78. Grade 12 students indicated a lower mean figure (3.37) for parental support than Grade 8 students (3.92).

Detailed information may be obtained by examining the pattern of results on the nine items in the scale, shown in Figure 7-6.

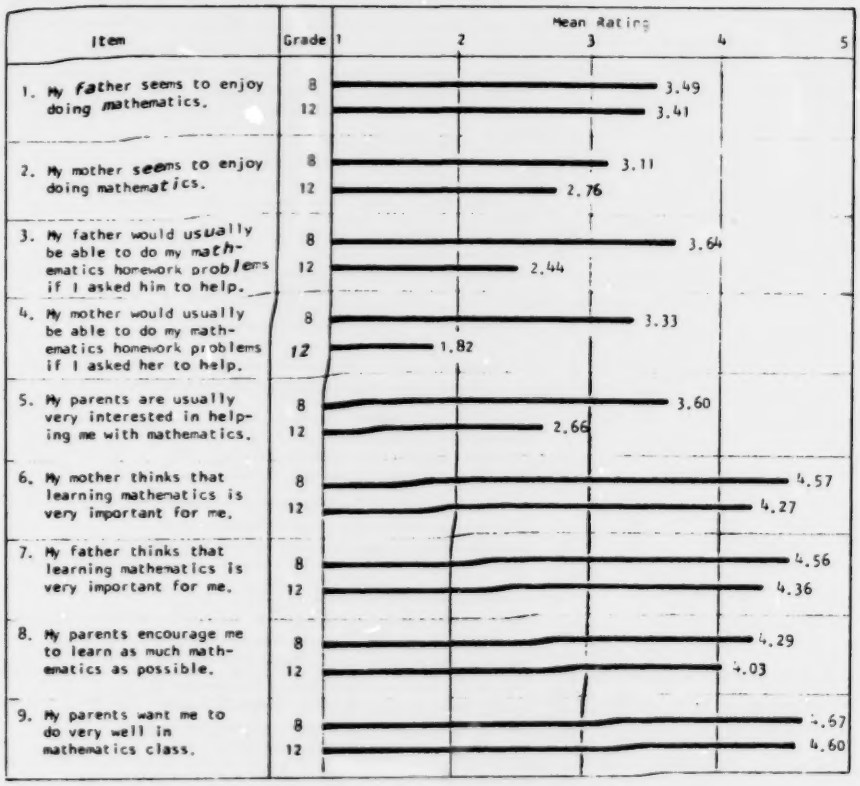


Figure 7-6. Item results for Parental Support for Mathematics.

Grade 12 students indicated less favorable responses than Grade 8 students on every item. The most marked difference was on item 4, which asked whether the mother would be able to do the student's homework. This result is not surprising since many studies have shown significant differences between the number of males and females studying mathematics at the upper grade levels in high school. In comparing father and mother, at both grade levels, father was perceived as enjoying mathematics more and being more competent in mathematics. The parents were seen as equal in their rating of the importance of mathematics for the student. Students gave both parents very high ratings on their desire to see the students do well in mathematics class.

7.5 Mathematics and Myself

The 19 items making up this scale were designed to determine how students react personally to the study of mathematics. They measured the extent to which students think mathematics is enjoyable, and to which they feel competent in doing mathematics. Several items provided measures of what has popularly been termed "mathematics anxiety": 15, 16, 17, 18, 19. For all items the rating scale was from 1 (Strongly negative) to 5 (Strongly positive).

Grade 12 students, with a mean of 3.51 and S.D. of 0.61, showed a slightly more positive outlook toward their relationship with mathematics and they were more consistent in this view than the Grade 8 students, who had a mean of 3.31, and a S.D. of 0.76. Reliabilities were very high, 0.92 and 0.90, as they were in the international pilot studies used to determine suitable items for the scale.

Individual items results are shown in Figure 7-7. Grade 12 students showed a more favorable response than Grade 8 students on 14 items; the two groups were roughly equal on the remaining five items. Differences were greatest on item 6: I am not so good at mathematics. This difference may reflect reality since the Grade 12 students have passed through the sieving process in reaching Grade 12 and have self-selected the mathematics course. Item 19, a measure of bewilderment in problem-solving, produced the lowest scores at both grade levels, indicating that students feel frustrated when unsuccessful in solving problems. Perhaps related to this is the response pattern to item 15, Working with numbers makes me happy, which students at both grade levels rated below the midpoint of the scale. Only 4% of the Grade 12 students strongly agreed with this statement.

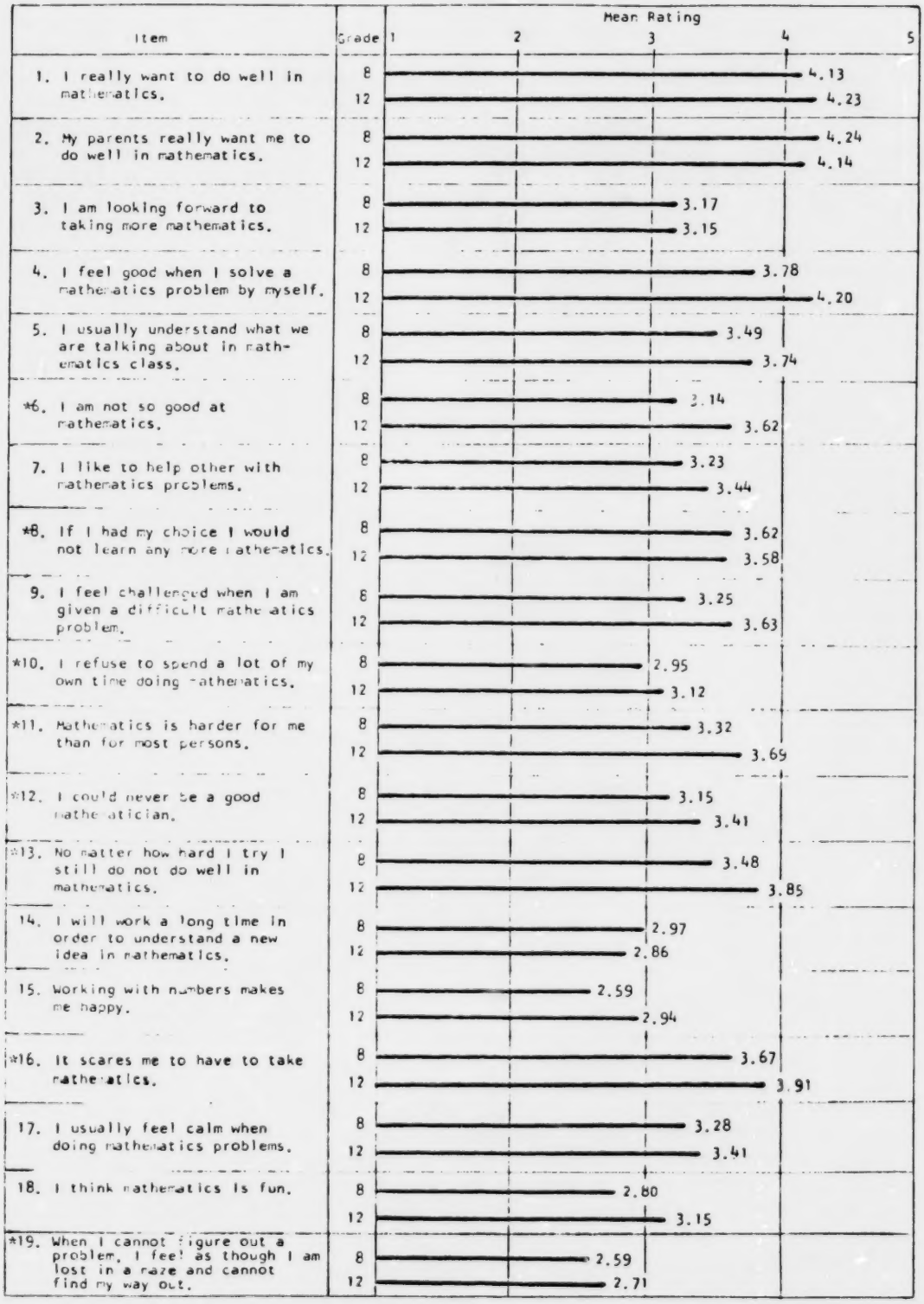

Item
Grade
Mean Rating
1
2
3
4
5
1. I really want to do well in mathematics.
8 4.13
12 4.23
2. My parents really want me to do well in mathematics.
8 4.24
12 4.14
3. I am looking forward to taking more mathematics.
8 3.17
12 3.15
4. I feel good when I solve a mathematics problem by myself.
8 3.78
12 4.20
5. I usually understand what we are talking about in mathematics class.
8 3.49
12 3.74
*6. I am not so good at mathematics.
8 3.14
12 3.62
7. I like to help other with mathematics problems.
8 3.23
12 3.44
*8. If I had my choice I would not learn any more mathematics.
8 3.62
12 3.58
9. I feel challenged when I am given a difficult mathematics problem.
8 3.25
12 3.63
*10. I refuse to spend a lot of my own time doing mathematics.
8 2.95
12 3.12
*11. Mathematics is harder for me than for most persons.
8 3.32
12 3.69
*12. I could never be a good mathematician.
8 3.15
12 3.41
*13. No matter how hard I try I still do not do well in mathematics.
8 3.48
12 3.85
14. I will work a long time in order to understand a new idea in mathematics.
8 2.97
12 2.86
15. Working with numbers makes me happy.
8 2.59
12 2.94
*16. It scares me to have to take mathematics.
8 3.67
12 3.91
17. I usually feel calm when doing mathematics problems.
8 3.28
12 3.41
18. I think mathematics is fun.
8 2.80
12 3.15
*19. When I cannot figure out a problem, I feel as though I am lost in a maze and cannot find my way out.
8 2.59
12 2.71

Figure 7-7. Item results for Mathematics and Myself.

On a more positive note, the vast majority of the students claimed they wanted to do well in mathematics and that their parents also wanted them to do well. The latter item corroborates the responses to a similar item in the Parental Support for Mathematics scale. Other items on which high scores were indicated, particularly at the Grade 12 level, were 4, 13, and 16. Taken as a group they indicate strong feelings of satisfaction with success in problem-solving, a feeling of confidence in ability to do mathematics, and a lack of fear of the subject.

7.6 Mathematics in Society

The eight items in this scale were designed to determine how students view mathematics in terms of its practical value. Two particular topics were built into this scale: the importance of mathematics in preparing for an occupation, and the usefulness of mathematics in everyday life.

There was no difference in ratings between the grade levels, each having a mean of 3.43. However, Grade 12 students were more consistent in their outlook with a S.D. of 0.65, as compared to the Grade 8 S.D. of 0.81. Reliabilities were very high, 0.88 and 0.82, considering the small number of items used. However, as with the other scales, all of the items in the scale appeared together in the questionnaire. A superficial consistency of responses could have been produced if students simply referred to their answers to other questions on the scale. The particulars of the breakdown of the entire collection of items into scales were not known at the time students responded to the questionnaire. Hence this feature may have inflated reliabilities across all scales.

Item results are shown in Figure 7-8. With the exception of two items, responses were similar between and within grades. On item 1, fully 32% of the Grade 8 students strongly agreed that it is important to know mathematics in order to get a good job, and 47% of the Grade 8 students agreed with the item. These figures are 11% and 50% respectively at the Grade 12 level. The 32% figure is remarkable since, in general on this scale, students seemed reluctant to respond at either extreme, that is, either to agree strongly or to disagree strongly. Two possibilities come to mind. It may be that teachers and parents have done extraordinarily well selling the notion that mathematics is a pre-requisite for gainful employment. On the other hand, it may be that Grade 8 students view mathematics as arithmetic, and a fundamental area of knowledge. This conjecture is supported to a degree by the fact that Grade 8 students tended to respond at the extremes on this scale

more readily than Grade 12 students. The latter may view mathematics as a more formal pursuit, algebraic or geometric, and may see less need for such esoterica in their occupations.

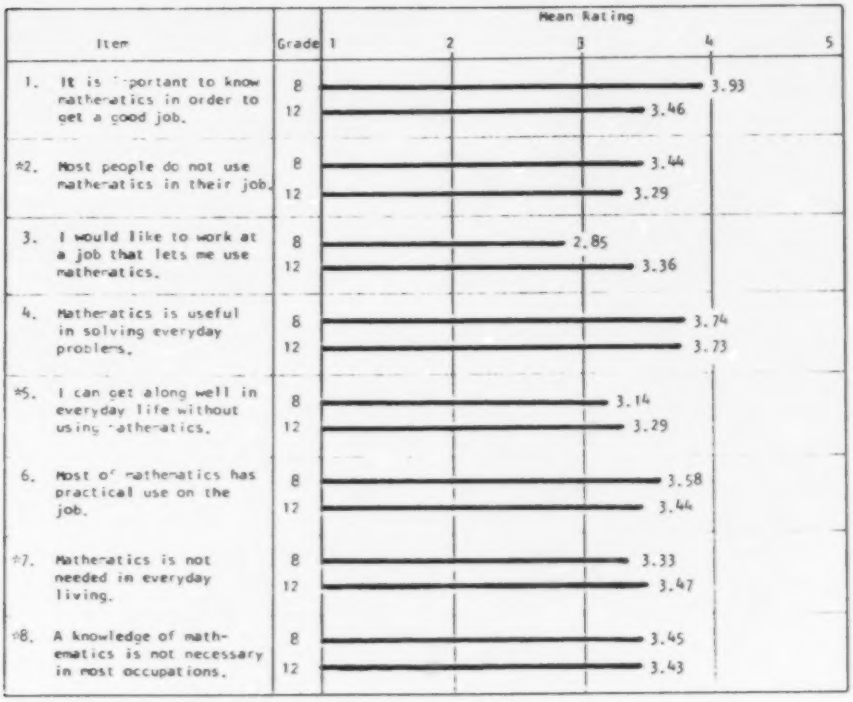


Figure 7-8. Item results for Mathematics in Society.

The second item on which different attitudes were indicated is item 3. Forty-five percent of the Grade 12 students would like to work at a job that requires mathematics, compared to 27% of the Grade 8 students. Again this may simply reflect the sorting mechanism under which only 38% of Grade 12 students study Grade 12 Algebra.

7.7 Mathematics and Gender

Four items were included to determine whether students felt there were sex differences in mathematical ability and in the need to know mathematics for career purposes. Essentially there was no difference between grade levels,

each showing means of about 3.83, indicating a high degree of support for equality of males and females. On the other hand, standard deviations of 1.09 and 1.03 represented a larger than usual spread of scores. A point of view tended to be held consistently across items, since at each grade level approximately one-quarter of the students scored at the maximum. Item means are shown in Figure 7-9.

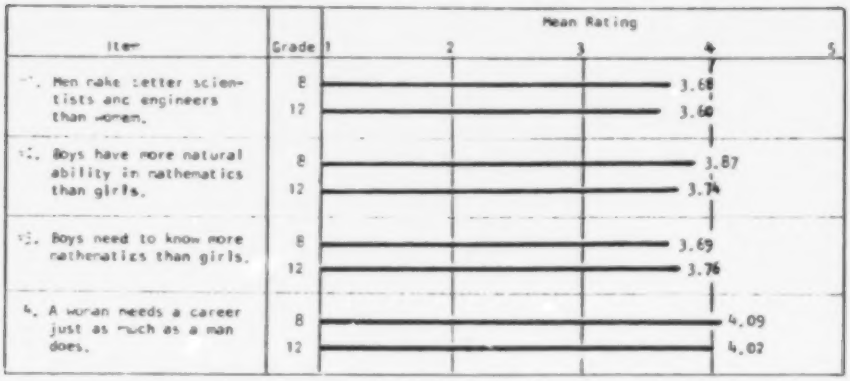


Figure 7-9. Item results for Mathematics and Gender.

7.8 Calculators and Computers

Eight items measured attitudes toward calculators and computers, four on each topic. Grade 12 students were somewhat favorably disposed with a mean of 3.47. The mean for students at the Grade 8 level was 3.06, indicating a rather noncommital attitude toward the merits of the two devices. From the item characteristics shown in Figure 7-10, this seems to be particularly true for calculators, the first four items on the scale.

Students generally were more positive about the efficacy and benefits of computers. Over two-thirds at both grade levels think that everyone should know something about computers. One wonders whether the 39% of the Grade 8 students who felt that using computers makes learning mathematics more mechanical and boring were simply speculating or whether they had been exposed to poor examples of computer aided instruction.

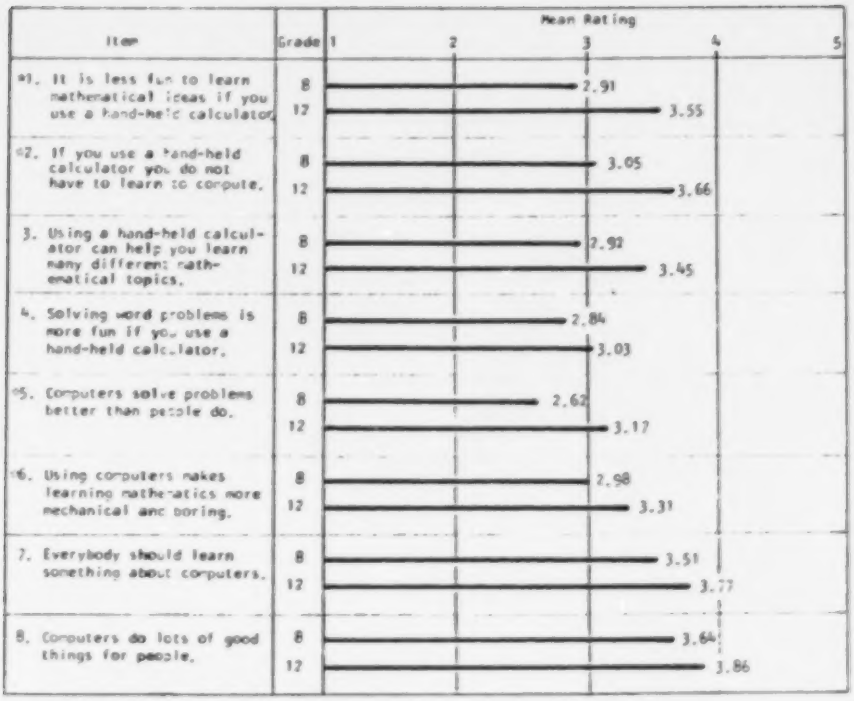


Figure 7-10. Item results for Calculators and Computers.

Item 5 was troublesome at each level, perhaps due to differing interpretations. Correlations between responses on this item and total scale responses were low.

The positive attitude of Grade 12 students toward calculators is encouraging, particularly in view of the feeling of over two-thirds of them that using a calculator does not relieve one of the task of learning to compute. This, along with the response patterns in items 1 and 3, and the favorable response indicated in the Mathematics in School section, suggests that students have had positive experiences with calculators in their high school years.

CHAPTER 8

MAJOR FINDINGS AND IMPLICATIONS

The results of the Second International Mathematics Study, both local and international, are an important source of information for all those in British Columbia who are conerned with the continued improvement of the teaching and learning of mathematics at the secondary school level. Over the next few years, as the international volumes are published and as follow-up analyses of the data are conducted, this pool of information will continue to grow. In a very real sense, therefore, this report is only a first step in the process of disseminating the results of the study.

Since British Columbia was one of the first of the participants in the international study to complete its data collection and publish a local report, few international results are available for purposes of comparison, except in the area of curriculum analysis. In addition, there has been no attempt made as yet to conduct analyses of the results to examine the relationships that may exist between the methods and resources used by teachers of mathematics and the achievement of their students.

In spite of such limitations of the scope of the report, it should be a valuable source of information for teachers of mathematics, administrators, researchers, and decision-makers. In particular, the information presented here should be extremely useful to all those involved in the forthcoming review and possible revision of the mathematics curriculum in this province.

In this concluding chapter, an attempt has been made to select from the vast number of pieces of information and results obtained in the study, those that are in some way most pertinent to the process of curriculum revision as well as those that are inherently interesting. As has been the case throughout the study and in this report, major emphasis has been given to the discussion of the curriculum and the teaching of mathematics rather than to achievement results.

Curriculum Analysis

- At the Population A (Grade 8) level, virtually all students in all of the participating countries are still studying mathematics. Moreover, the B. C. curriculum at that level coincides fairly well with the international grid. In other words, the B. C. mathematics curriculum at the Grade 8 level is similar in many respects to the curricula of other countries.

- In some countries, only a very small proportion of the age cohort is still studying mathematics in the final year of secondary school; less than 10% in some cases. In British Columbia, for the Algebra 12 course, this figure is just over 30%. This fact has obvious implications for the content of the Algebra 12 course, since it must be made suitable for a comparatively large proportion of the student population.

- From the data obtained regarding the amount of time available in the school program for mathematics, it is clear that, in many countries, students at the senior secondary level are able to specialize in mathematics to a much greater degree than would be possible in British Columbia.

- A major difference between the Algebra 12 curriculum in British Columbia and the comparable curricula in other places is the importance given to the study of calculus. In B. C., calculus is not part of the Algebra 12 curriculum, except as one of a sequence of optional topics for the enriched format of the course. On the other hand, the study of calculus is a major component of the mathematics curriculum at the senior secondary level in other countries, notably in Europe.

- At both levels in British Columbia, little or no importance is given to the study of probability and statistics. This contrasts sharply with the importance given to them at either or both levels in most other countries. More importantly, given the importance of these topics in contemporary society, this situation should be examined and discussed as soon as possible.

The Teaching of Mathematics in Grade 8

- Over 20% of Grade 8 teachers teach students the British system of measurement as well as the metric system, in spite of the fact that the Curriculum Guide calls for only the latter to be taught, and that the prescribed textbooks refer only to the metric system. It may be that teachers have de-

cided that this generation of students needs to be familiar with both systems, and, therefore, both systems need to be taught. The fact that the United States now appears to be farther away from converting to the metric system than at any other time in the recent past may lend additional weight to such an opinion, and this matter should be discussed as part of the curriculum renewal process.

- Teachers rarely assume that students have mastered concepts or skills in previous grades to the point where no re-teaching or review is needed. Even topics such as the concept of a fraction, which is first introduced in the primary grades, were reported as having been taught or reviewed in almost 90% of classrooms. This may be due, in part at least, to a lack of familiarity on the part of Grade 8 teachers with the content of the mathematics curriculum at the elementary school level.

- There is a substantial lack of agreement among teachers about which formulas for the area and volume of various two- and three-dimensional figures students are expected to be familiar with by the end of Grade 8.

- There are indications that in teaching computational techniques, such as multiplication of decimals, teachers emphasize the sequence of steps to be followed in using the algorithm rather than an understanding of why the algorithm works the way it does. For example, 98% of teachers agreed that they taught students how to multiply decimals by referring back to multiplication of whole numbers and giving students a rule for placing the decimal point in the answer. Similar, rule-oriented approaches to teaching were apparent for topics such as operations with positive and negative numbers, and the development of formulas.

- Although virtually all teachers reported spending some time on each of the five major areas of the mathematics curriculum covered in the five Classroom Process questionnaires, they differed significantly on the amount of time they devoted to them. For example, 10% of teachers reported spending at most eight class periods in the topic of integers while another 10% spent over twenty periods on the same topic.

- In the opinion of teachers, the principal factors which might account for a lack of satisfactory academic progress in mathematics by students were students' indifference, absenteeism, or lack of ability. Few felt that unsatisfactory progress could be attributed to a shortage of appropriate resources for teaching or to a lack of expertise on the part of teachers.

- According to teachers, whether or not a given approach to the teaching of a particular concept or skill would be easy for students to understand was the single most influential factor in determining which of a number of alternative teaching approaches to use. This factor was substantially more important than whether or not students would enjoy the approach or whether that approach was used in the textbook. Analagously, the fact that a given apprach was seen as being difficult for students to understand was a powerful factor in decisions not to use a particular teaching approach.

- While teachers did not select teaching approaches primarily because they were present in the textbook, many indicated that they would be reluctant to follow approaches other than the ones in the textbook. This would seem to indicate that, if certain topics, approaches, or orientations are to be implemented in the mathematics curiculum, care must be taken to ensure that these are included either in the prescribed textbooks or in other instructional materials which are made available to teachers.

- The majority of teachers emphasized only one approach to the teaching of a given topic. While they might have used others or referred to them occasionally, they tended to emphasize only one.

- Over 25% of Grade 8 teachers of mathematics reported that, in their teaching, they seldom followed the treatment given to a topic in the textbook. Since, a large number also reported that they relied primarily on locally-developed worksheets, it may be the case that many teachers are not utilizing the prescribed textbooks to any significant degree.

- Although most Grade 8 teachers agreed that students should be permitted to use calculators, they were also strongly of the opinion that students should have a high degree of mastery of computational skills.

The Teaching of Mathematics in Grade 12

- There are very substantial differences in the amounts of time teachers spend on the different components of the Algebra 12 course. For analytic geometry, the range was from 6 to 45 periods, with 21 periods as the mean. In the case of trigonometry, the range was from 10 to 56 periods, with the mean being 29. For algebra, the range was from 7 to 96 periods, with the mean being 49.

- Approximately 40% of Algebra 12 teachers agreed with a statement to the effect that there should be a separate, semester-long course in trigonometry. Only 3% disagreed with this opinion; the remainder were undecided.

- As in the case of Grade 8, Algebra 12 teachers spend substantial amounts of time reviewing material that was covered in earlier courses.

- Relatively little time appears to be spent on developing students' ability to construct graphs of functions.

- Despite the fact that virtually all Algebra 12 teachers permitted their students to use calculators, 50% of them emphasized the computational applications of logarithms in their teaching.

- In presenting various formulas, identities, or generalizations to their students, almost all of the Algebra 12 teachers provided some form of proof or informal justification. However, they seldom expected their students to be able either to replicate such proofs or to produce them on their own. Instead, they expected students to recall the formulas and apply them.

- The use of calculators in class was permitted by 97% of Algebra 12 teachers. Moreover, 16% of the teachers said that they also made use of microcomputers in some manner.

- Results indicate that Algebra 12 teachers adhere more closely to the course outline as contained in the Curriculum Guide than do teachers of Mathematics 8. This may well be due to the fact that the course outline for Algebra 12 is much more detailed than the one for Mathematics 8.

- Algebra 12 teachers are more likely to use a particular teaching approach because they feel it will be easy for students to understand than because they think students will enjoy it. Analagously, the fact that a given approach is seen as being difficult is a stronger factor in deciding not to use a given approach to a topic than is the belief that students will not enjoy it.

- In a typical Algebra 12 class, correction of homework occupies 25% of the time, lecturing by the teacher an additional 40%, and individual seat work by students, 20%. Little time is allocated for having students work together in small groups.

- Results show that Algebra 12 teachers typically do not differentiate between levels of ability in their teaching. Most teachers teach the same content in the same way and give the

same assignments to all students.

Achievement Results--Grade 8

- Over 80% of the test items developed for use in the Second International Mathematics Study were judged to be appropriate for B. C. students in Grade 8. Students were told that the tests contained some items with which they would not be familiar, and they were advised to skip those items.

- All Grade 8 students were administered a forty-item test at the beginning of the school year, and the same test was re-administered near the end of the course. Results on the posttest were 4.4 points higher than on the pretest, an improvement of about 25%.

- The range of growth scores on the forty-item core test was from -1.8 to 10.1. That is, in one class the posttest score was 1.8 points lower than the pretest score, and, at the other extreme, the performance in one class improved by 10.1 points.

- Gains in achievement from pretest to posttest were greatest on the algebra and geometry sub-tests, the two areas which consisted primarily of content presented for the first time in Grade 8.

- In a number of cases, classes performed more poorly on one or more of the sub-tests on the posttest than on the pretest. These losses occurred mainly on those subtests dealing with content from earlier grades, for example, fractions, measurement, and ratio and proportion.

- Teachers were asked to indicate whether or not the mathematics content required to respond to each item had been presented to students. Overall, there was only a moderate correlation between this Opportunity-to-Learn measure and the achievement results. This may mean that teachers tend to underestimate their students' knowledge or that they are somewhat unfamiliar with the curriculum of earlier grades. Alternatively, it may be that teachers did not interpret the Opportunity-to-Learn question in the way that had been intended. Further analyses of these results should clarify this issue.

Achievement Results--Grade 12

- No pretesting was done with Grade 12 students. Moreover, since the test items were distributed among eight rotated forms, there was no common set of items which all students wrote.

- Of the total of 136 items on the eight test forms, 45 were judged to be inappropriate for B. C. students because they dealt with content outside the Algebra 12 curriculum.

- In constructing items for the tests, four levels of cognitive behavior were used: computation, comprehension, application, and analysis. In general, results show that achievement decreases as cognitive level increases. Similar results were found at the Grade 8 level.

- On the 91 items which were judged to be appropriate for the B. C. curriculum, the average percent correct was about 42%. Given the level of difficulty of the items, this would appear to be a satisfactory result.

Students' Attitudes

- When asked to react to a list of fifteen topics and activities in mathematics, students at both the Grade 8 and Grade 12 levels indicated that they enjoyed them, felt they were important, and did not find them to be too difficult.

- Teachers gave approximately the same ratings as their students to most of the items. Among the exceptions to this finding were that teachers felt that problem solving was more important, more enjoyable, and more difficult than their students did.

- Teachers and students both agreed that mathematics is a dynamic, growing field in which new developments are continually being made and not a static, unchanging one. Teachers were more positive in this regard than their students, and Grade 12 students were somewhat more positive than those in Grade 8.

- Students indicated that their parents considered mathematics to be an important area of study, and that their parents wanted them to do well in mathematics. Students at both levels reported that, while both parents encouraged them to do well in mathematics and believed that learning mathematics was important, their fathers enjoyed mathematics more than their mothers and were more likely to be able to assist

them with assignments.

- Overall, Algebra 12 students' attitudes toward mathematics were slightly more positive than those of Grade 8 students.

- A clear majority of students at both levels agreed that it was important to know mathematics in order to get a good job; indeed, 32% of Grade 8 students strongly agreed with that opinion.

- On a sub-scale consisting of four items, Grade 8 and Grade 12 students indicated that both males and females were equally capable in mathematics, had the same need for careers, and had the same need to know mathematics.

- Over two-thirds of students at both levels were of the opinion that it was important for them to know something about computers, and that computers could be used in beneficial ways.

APPENDIX A

AUTHORS OF THE REPORT

Michael Dirks is a doctoral student in mathematics education at the University of British Columbia. He has taught extensively in the United States and in Thailand.

Thomas O'Shea is Assistant Professor of Mathematics Education at Simon Fraser University. He was a member of the Contract Team for the 1981 Mathematics Assessment and served as research assistant for the Mathemtics Achievement Test Project.

David F. Robitaille is Professor of Mathematics Education at the University of British Columbia. He served as the Research Coordinator for the Second International Mathematics Study in B. C., and as chairman of the Contract Teams for both the 1977 and 1981 Mathematics Assessments.

APPENDIX B

PARTICIPATING TEACHERS - GRADE 8

The following teachers made a significant contribution to the study through both the administration of test items and completion of a series of Classroom Process Questionnaires.

POPULATION A - GRADE 8 MATHEMATICS

ZONE 1 (OKANAGAN)

Teacher	School	District
D. Joba	W. L. Seaton	Vernon
D. Pazlis	100 Mile	Cariboo-Chilcotin
A. Cullum	Revelstoke	Revelstoke
F. Jones	Rutland	Central Okanagan
B. McDonald	Sa-Hali	Kamloops
B. Bosman	Princess Margaret	Penticton
B. Heckrodt	Shuswap	Shuswap
W. Waddell	KLO	Central Okanagan
K. Williamson	Mount Boucherie	Central Okanagan
A. Bill	Princeton	Princeton
G. Bateman	A. Fortune	Shuswap
M. Hepples	Brockhurst	Kamloops
G. Tobias	North Shuswap	Shuswap
J. Young	Nicola Valley	Merritt

ZONE 2 (METRO)

W. Woods	New Westminster	New Westminster
A. Lazzarotto	Kensington	Burnaby
I. Hooper	Gladstone	Vancouver
D. Sumas	Sir F. Banting	Coquitlam
B. Peacock	Sands	Delta
G. Davies	Sir W. Churchill	Vancouver
R. Hundvik	David Thompson	Vancouver
H. Kost	McPherson Park	Burnaby
J. Maddison	Dr. C. Best	Coquitlam
R. Umemera	Kitsilano	Vancouver
D. Tachiyama	White Rock	Surrey
B. Lohner	Johnston Heights	Surrey
D. Sinclair	Moody	Coquitlam
H. Franklin	Magee	Vancouver
C. McKillop	L. Shepherd	Surrey
I. Peters	Balmoral	North Vancouver
C. Simister	Earl Marriott	Surrey
G. Thompson	West Whalley	Surrey
D. McClenahan	Sentinel	West Vancouver
P. Yates	Royal Oak	Burnaby

ZONE 2 (METRO) cont'd

Teacher	School	District
D. Gordon	Seycove Comm.	North Vancouver
R. Picketti	Tsawassen	Delta
G. Smedley	Killarney	Vancouver
S. Kennett	John Oliver	Vancouver
G. Maynard	Prince of Wales	Vancouver

ZONE 3 (FRASER VALLEY)

J. Tucker	Abbotsford	Abbotsford
S. Peterson	Clearbrook	Abbotsford
G. Enns	Brooksford	Langley
B. Westby	Vedder	Chilliwack
S. Moore	Aldergrove	Langley
W. Stewart	Oceanview	Powell River
A. Bakken	A. D. Rundle	Chilliwack
L. Mjanes	Chatelech	Sunshine Coast
H. Voth	Sardis	Chilliwack
E. Chumley	Pemberton	Howe Sound
S. Leckie	Mission	Mission

ZONE 4 (ISLAND)

A. Avery	Lansdowne	Greater Victoria
R. Sandberg	E. J. Dunn	Alberni
R. Faught	Quamichan	Cowichan
L. Klassen	Campbell River	Campbell River
D. Zozula	Oak Bay	Greater Victoria
W. Dale	Courtney	Courtenay
R. Bustart	Lake Trail	Courtenay
K. Cox	Qualicum Beach	Qualicum
E. Petticrew	Dunsmuir	Sooke
A. Wise	S. Gordon	Lake Cowichan
R. Dunn	Gulf Islands	Gulf Islands
A. Rundlecock	Pender Island	Gulf Islands
B. Farquarson	Ladysmith	Nanaimo
B. Edwards	Comox	Courtenay

ZONE 5 (KOOTENAYS)

R. Ferguson	Laurie	Cranbrook
C. Johnston	Parkland	Cranbrook
P. Gerber	McKim	Kimberley
P. Zaatsoff	Beaver Valley	Trail
D. Grimshire	Nakusp	Arrow Lakes
R. Dean	Mt. Sentinel	Nelson
D. Tambellini	Grand Forks	Grand Forks

ZONE 6 (NORTH)

Teacher	School	District
A. Klaan	Mt. Elizabeth	Kitimat
J. Whyte	Booth Memorial	Prince Rupert
G. Shea	College Heights	Prince George
D. Watt	Duchess Park	Prince George
J. Walden	Bert Bowes	Peace River North
S. Cairns	John McInnis	Prince George
M. Benner	Houston	Smithers
G. White	Chandler Park	Smithers
B. Tedesco	Kelly Road	Prince George
B. Bateman	Blackburn	Prince George
P. Howe	Grassy Plains	Burns Lake

APPENDIX C

PARTICIPATING TEACHERS - GRADE 12

POPULATION B - ALGEBRA 12

ZONE 1 (OKANAGAN)

R. Niehaus	Vernon	Vernon
R. Hawkins	Vernon	Vernon
H. Dunsmore	Kamloops	Kamloops
W. Epp	Kamloops	Kamloops
L. Pada	Kelowna	Central Okanagan
R. McKenzie	Kelowna	Central Okanagan
M. Cianci	Norkam	Kamloops
J. Derban	Norkam	Kamloops
R. Marriott	Salmon Arm	Shuswap
J. Sowinski	Okanagan Mission	Central Okanagan
A. Swindells	Springvally	Central Okanagan
R. Norman	KLO	Central Okanagan
G. Davis	George Elliott	Central Okanagan
B. Doyle	Chase	Kamloops
G. Banting	Kumsheen	South Caribɔo

ZONE 2 (METRO)

B. Jordan	Centennial	Coquitlam
K. Murray	Centennial	Coquitlam
J. Bianco	Centennial	Coquitlam
K. Mayson	Carson Graham	North Vancouver
A. Vickers	Carson Graham	North Vancouver
D. Clague	Carson Graham	North Vancouver
D. Spencer	Steveston	Richmond
T. Kagetsu	Steveston	Richmond
R. Charlesworth	Burnaby South	Burnaby
B. Izen	Burnaby South	Burnaby
T. Dixon	Port Coquitlam	Coquitlam
H. Enns	Port Coquitlam	Coquitlam

ZONE 2 (METRO) cont'd

Teacher	School	District
A. Shaw	Burnaby Central	Burnaby
S. Sutcliffe	Burnaby Central	Burnaby
P. Burke	Richmond	Richmond
J. Hope	North Delta	Delta
J. Schellenberg	John Oliver	Vancouver
A. Olsen	New Westminster	New Westminster
R. Michelson	Queen Elizabeth	Surrey
J. Arnason	Semiahmoo	Surrey
B. Bowles	Windsor	North Vancouver
K. Sandercock	Windsor	North Vancouver
B. Street	Burnaby North	Burnaby
E. Whitney	Gladstone	Vancouver
B. Kokoskin	Handsworth	North Vancouver
D. Ellis	Sir W. Churchill	Vancouver
F. Maclean	Vancouver Tech.	Vancouver
A. Hattrick	Point Grey	Vancouver
B. Hall	Sentinel	West Vancouver
C. Rudolph	David Thompson	Vancouver
R. McTaggart	Kitsilano	Vancouver
K. Hagg	Sir C. Tupper	Vancouver
R. West	Windermere	Vancouver
H. Turnbull	Alpha	Burnaby
G. Semail	Lord Byng	Vancouver
M. Lessey	Princess Margaret	Surrey
G. Burden	South Delta	Delta
H. Janzen	Seaquam	Delta
G. Mitchell	Lord Tweedsmuir	Surrey
G. Burhoe	Delta	Delta
R. Galloway	King George	Vancouver
J. Wood	Killarney	Vancouver
D. Alvaro	Argyle	North Vancouver

ZONE 3 (FRASER VALLEY)

G. Horner	Abbotsford	Abbotsford
G. Nachtigal	Chilliwack	Chilliwack
D. Kennedy	Langley	Langley
J. Blackhall	Max Cameron	Powell River
R. Craven	W. J. Mouat	Abbotsford
H. Brigden	Garibaldi	Maple Ridge

ZONE 4 (ISLAND)

G. Booth	Nanaimo District	Nanaimo
P. Koski	Mount Douglas	Greater Victoria
J. Marsh	Cowichan	Cowichan
R. Hudstith	Cowichan	Cowichan
V. Thompson	G. P. Vanier	Courtenay
E. Mahrt	G. P. Vanier	Courtenay

ZONE 4 (ISLAND) cont'd

Teacher	School	District
R. Tschritter	Oak Bay	Greater Victoria
K. Newell	Victoria	Greater Victoria
J. Chell	Victoria	Greater Victoria
C. Waldman	Reynolds	Greater Victoria
B. Ball	Ballenas	Qualicum
J. Clemens	Spectrum Comm.	Greater Victoria
J. Runke	Ladysmith	Nanaimo
H. Archer	Gulf Islands	Gulf Islands
C. Zacharias	North Island	Vancouver Is. North

ZONE 5 (KOOTENAYS)

G. Mitchell	Mount Baker	Cranbrook
G. De Vries	L. V. Rogers	Nelson
F. Perehudoff	Stanley Humphreys	Castlegar
C. Gris	Prince Charles	Creston-Kaslo
P. Miller	Golden	Golden
H. Robinson	Sparwood	Fernie
E. Knecht	Nakusp	Arrow Lakes

ZONE 6 (NORTH)

W. Hanik	Prince George	Prince George
P. Gupta	Prince George	Prince George
D. Angus	North Peace	Peace River North
P. Neave	Correlieu	Quesnel
K. Savage	Mount Elizabeth	Kitimat
K. Fraser	South Peace	Peace River South
G. Dowles	Smithers	Smithers
S. Brar	Mackenzie	Prince George
B. Poditz	Valemont	Prince George

APPENDIX D

SAMPLE QUESTIONNAIRE - GRADE 8

INTERNATIONAL ASSOCIATION for the
EVALUATION of EDUCATIONAL ACHIEVEMENT

SECOND
Study of
MATHEMATICS

GRADE 8
TOPIC SPECIFIC QUESTIONNAIRE
ALGEBRA (FORMULAS AND EQUATIONS) AND INTEGERS

(Booklets 14L and 15L Combined)

FOR NATIONAL CENTRE USE ONLY

PROVINCE OF BRITISH COLUMBIA
MINISTRY OF EDUCATION
DIVISION OF PUBLIC INSTRUCTION
LEARNING ASSESSMENT BRANCH

Check here if none of integers (positive and negative whole numbers), formulae or equations are included in your program. In that case, disregard the remainder of the questionnaire and return it to B.C. Research in the envelope provided. ___1

CHECK the response which best describes the use you made of each of the following materials in your instruction on integers, formulae and equations.

RESPONSE CODES:

1. *primary source, used frequently*
2. *secondary source, used occasionally*
3. *not used or rarely used*

1.	School Mathematics II (Addison-Wesley)	___1	___2	___3
2.	Mathematics II (Ginn)	___1	___2	___3
3.	Essentials of Mathematics II (Ginn)	___1	___2	___3
4.	Other published text materials (e.g., textbooks, workbooks, and worksheets)	___1	___2	___3
5.	Locally produced text materials (e.g., textbooks, workbooks, or worksheets)	___1	___2	___3
6.	Commercially or locally produced individualized materials (e.g., programmed instruction or computer assisted instruction)	___1	___2	___3
7.	Commercially or locally produced films, filmstrips, or teacher demonstration models	___1	___2	___3
8.	Commercially or locally produced laboratory materials for student use (e.g., games or manipulatives)	___1	___2	___3

TEACHING TOPICS

The topics given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. *taught as new content*
2. *reviewed and then extended*
3. *reviewed only*
4. *assumed as prerequisite knowledge and neither taught nor reviewed*
5. *not taught and not assumed as prerequisite knowledge*

Integers

9.	The concept of positive and negative integers.	___1	___2	___3	___4	___5
10.	Addition of integers (+ and -).	___1	___2	___3	___4	___5
11.	Subtraction of integers (+ and -).	___1	___2	___3	___4	___5
12.	Multiplication of integers (+ and -).	___1	___2	___3	___4	___5
13.	Division of integers (+ and -).	___1	___2	___3	___4	___5
14.	Structural properties of the set of integers (e.g., commutativity, associativity, distributivity, etc.)	___1	___2	___3	___4	___5
15.	Order relations in the set of integers.	___1	___2	___3	___4	___5

Formulas and Equations

16.	Evaluation of formulas for given values of the variables. EX: Given A = L x W. If L = 4 and W = 5, substitute for L and W and find the value of A.	___1	___2	___3	___4	___5
17.	Deriving formulas or equations. EX: Each weight stretches a spring 3 cm. What formula gives the stretch (total) for n weights?	___1	___2	___3	___4	___5
18.	Solving literal equations. EX: Solve $Y = \frac{2x + r}{z}$ for r.	___1	___2	___3	___4	___5
19.	Solving linear equations. EX: Solve 4x - 3 = 19	___1	___2	___3	___4	___5

The interpretations of integers given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this interpretation in the students' text?

If you EMPHASIZED the given interpretation (i.e. if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given interpretation (i.e. if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

			1st Primary Reason	2nd Primary Reason	1st Primary Reason	2nd Primary Reason
20. **Extending the number ray to the number line:**						
I extended the number ray (0 and positive numbers) to the left by introducing direction as well as magnitude.	Yes	___1				
	No	___2				
Ex: ---------- -4 -3 -2 -1 0 1 2 3 4 -3 means 3 units to the left of 0.						
___1 ___2 ___3			☐	☐	☐	☐
21. **Extending the number system to find solutions to equations:**						
I discussed the need to extend the positive integers in order to find a solution to equations like __ + 7 = 5.	Yes	___1				
	No	___2				
___1 ___2 ___3			☐	☐	☐	☐
22. **Using vectors or directed segments on the number line:**						
I defined an integer as a set of vectors (directed line segments) on the number line.	Yes	___1				
	No	___2				
Ex: -2 can be represented by any of: ← ← ← -10 -5 0 5 10						
Ex: +2 can be represented by any of: → → → -10 -5 0 5 10						
___1 ___2 ___3			☐	☐	☐	☐

The interpretations of integers given below may be included in your instructional program. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. *Emphasized (used as a primary explanation, referred to extensively or frequently)*
2. *Used but not emphasized*
3. *Not used*

Was this interpretation in the students' text?

If you EMPHASIZED the given interpretation (i.e. if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:

1. *Well known to me*
2. *In B.C. or local curriculum guide*
3. *Easy for students to understand*
4. *Enjoyed by students*
5. *Related to math of prior grades*
6. *Useful in math of later grades*
7. *I was taught it was appropriate*
8. *Emphasized in student text*

If you DID NOT USE the given interpretation (i.e. if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. *Not well known to me*
2. *Not in B.C. or local curriculum guide*
3. *Hard for students to understand*
4. *Disliked by students*
5. *Not related to math of prior grades*
6. *Not useful in math of later grades*
7. *I was taught it was inappropriate*
8. *Not emphasized in student text*

---

23. **Defining integers as equivalence classes of whole numbers:**

    I developed the integers as equivalence classes of ordered pairs of whole numbers.

    Yes ___1
    No ___2

    Ex: $((0,2),(1,3),(2,4),\ldots) = \bar{2}$

    or $((a,b) \in W \times W : b = a + 2) = \bar{2}$

    ___1 ___2 ___3

    1st Primary Reason ☐ 2nd Primary Reason ☐ 1st Primary Reason ☐ 2nd Primary Reason ☐

24. **Using examples of physical situations:**

    I developed integers by referring to different physical situations which can be described with integers.

    Yes ___1
    No ___2

    Ex: thermometer, elevation, money (credit/debit), sports (scoring), time (before/after), etc.

    ___1 ___2 ___3

    ☐ ☐ ☐ ☐

The procedures given below deal with the topic of addition of integers. CHECK the response code which describes the treatment of each procedure in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this procedure in the students' text?

If you EMPHASIZED the given procedure (i.e. if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given procedure (i.e. if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

	Was this procedure in the students' text?	1st Primary Reason	2nd Primary Reason	1st Primary Reason	2nd Primary Reason
25. Addition on the number line: I used the number line to add integers.	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐
26. Addition by rules: I used rules to add integers. Ex: If both addends have the same sign, the sum is found by adding their numerical (absolute) values and adjoining the common sign.	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐
27. Use of physical situations: I used physical situations to add integers. Ex: In climbing out of the Dead Sea Valley, the car started at an elevation of -643 feet and climbed 432 feet to an elevation of _____ feet.	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐

The procedures given below deal with the topic of subtraction of integers. CHECK the response code which describes the treatment of each procedure in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

If you EMPHASIZED the given procedure (i.e. if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given procedure (i.e. if you checked 3 in the first column) WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

	Was this procedure in the students' text?		If you EMPHASIZED: 1st Primary Reason	If you EMPHASIZED: 2nd Primary Reason	If you DID NOT USE: 1st Primary Reason	If you DID NOT USE: 2nd Primary Reason
28. Subtraction as addition of opposites: I used the number line to subtract integers by starting at the minuend and going the number of units indicated by the subtrahend but in the direction opposite of its sign. ___1 ___2 ___3	Yes ___1	No ___2	☐	☐	☐	☐
29. Subtraction as inverse of addition: I used the inverse relation between addition and subtraction to subtract integers. Ex: $^{+}4 - {}^{-}3 =$ ___ Solve $^{+}4 =$ ___ $+ {}^{-}3$ ___1 ___2 ___3	Yes ___1	No ___2	☐	☐	☐	☐
30. Subtraction by rules: I used rules to subtract integers. Ex: To subtract an integer, add its opposite. ___1 ___2 ___3	Yes ___1	No ___2	☐	☐	☐	☐

The procedures given below deal with the topic of subtraction of integers. CHECK the response code which describes the treatment of each procedure in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this procedure in the students' text?

If you EMPHASIZED the given procedure (i.e. if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given procedure (i.e. if you checked 3 in the first column), WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

	Was this procedure in the students' text?	1st Primary Reason	2nd Primary Reason	1st Primary Reason	2nd Primary Reason
31. Subtraction as a number of units: I extended the meaning of subtraction of whole numbers (i.e. y - x means the number of units from x to y) to integers. Ex: $^{+}4 - {}^{-}3$ means the number of units from $^{-}3$ to $^{+}4$.	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐
32. Subtraction as distance: I used the number line to subtract integers by finding the number of units (or distance) from the subtrahend to the minuend.	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐
33. Subtraction as "what must be added": I interpreted subtraction to mean "what must be added" to the subtrahend to get the minuend. Ex: $^{+}4 - {}^{-}3 =$ ___ means "what must be added to $^{-}3$ to get $^{+}4$"	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐

The following statements describe methods by which a teacher might develop the concept of the product of integers. CHECK the response code to indicate the extent to which that method of developing the concept was used with your class.

RESPONSE CODES:

1. *Emphasized (used as a primary method of development, referred to extensively or frequently)*
2. *Used, but not emphasized*
3. *Not used.*

34. Development by use of repeated addition:

I developed the concept of multiplication by appealing to repeated addition, e.g., $4 \times {}^{-}3 = {}^{-}3 + {}^{-}3 + {}^{-}3 + {}^{-}3 = {}^{-}12$ ____1 ____2 ____3

35. Development by the extension of properties of the whole number system:

I developed the concept of multiplication of integers by using the commutative, associative, and distributive properties to justify the products, e.g.,

${}^{-}4 \times {}^{-}3 =$ ____

But $0 = ({}^{-}4 + {}^{+}4) \times {}^{-}3$
$= ({}^{-}4 \times {}^{-}3) + ({}^{+}4 \times {}^{-}3)$
$= ({}^{-}4 \times {}^{-}3) + {}^{-}12$

Hence ${}^{-}4 \times {}^{-}3 = {}^{+}12$ ____1 ____2 ____3

36. Development by use of physical situations:

I developed the concept of multiplication of integers by appealing to physical situations that might illustrate the product of positive and negative numbers, e.g., A refrigerator is cooling at a rate of 4° per minute. Its thermometer is at 0°. What will be its temperature 4 minutes from now? ____1 ____2 ____3

37. Development by use of patterns:

I developed the concept of multiplication of integers by appealing to patterns of products, e.g.,

$+4 \times -3 = -12$
$+3 \times -3 = -9$
$+2 \times -3 = -6$
$+1 \times -3 = -3$
$0 \times -3 = 0$
$-1 \times -3 = 3$
$-2 \times -3 = 6$

___1 ___2 ___3

38. No development -- students were given rules:

I did not develop the facts for multiplication of integers by using any of the above methods. I instead gave them rules similar to the following.

If the signs are alike, the answer is positive. If they are different the answer is negative. If one factor is zero, the answer is zero.

___1 ___2 ___3

The procedures given below deal with methods for solving linear equations. CHECK the response code which describes the treatment of each procedure in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

Was this procedure in the students' text?

If you EMPHASIZED the given procedure (i.e. if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given procedure (i.e. if you checked 3 in the first column), WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

---

39. Using properties of equality with operations with numbers:

Ex: $7x + 5 = 40$
$7x + 5 - 5 = 40 - 5$
(Subtract 5 from both sides)
$7x = 35$
(arithmetic fact)
$\frac{7x}{7} = \frac{35}{7}$
(divide both sides by 7)
$x = 5$

Yes ___1
No ___2

___1 ___2 ___3

1st Primary Reason	2nd Primary Reason	1st Primary Reason	2nd Primary Reason
☐	☐	☐	☐

40. Using inverse operations with numbers:

Ex: $7x + 5 = 40$
$7x + 5 - {}^{-}5 = 40 + {}^{-}5$
(add the inverse of 5 to both sides)
$7x = 35$
$\frac{1}{7} \cdot (7x) = \frac{1}{7} \cdot 35$
(multiply both sides by the reciprocal of 7)
$x = 5$

Yes ___1
No ___2

___1 ___2 ___3

☐	☐	☐	☐

The procedures given below deal with methods for solving linear equations. CHECK the response code which describes the treatment of each procedure in your class.

RESPONSE CODES:

1. Emphasized (used as a primary explanation, referred to extensively or frequently)
2. Used but not emphasized
3. Not used

If you EMPHASIZED the given procedure (i.e. if you checked 1 in the first column) WRITE the numbers of the response codes which show the two primary reasons for its emphasis.

RESPONSE CODES:

1. Well known to me
2. In B.C. or local curriculum guide
3. Easy for students to understand
4. Enjoyed by students
5. Related to math of prior grades
6. Useful in math of later grades
7. I was taught it was appropriate
8. Emphasized in student text

If you DID NOT USE the given procedure (i.e. if you checked 3 in the first column), WRITE the numbers of the response codes which show the two primary reasons for not using it.

RESPONSE CODES:

1. Not well known to me
2. Not in B.C. or local curriculum guide
3. Hard for students to understand
4. Disliked by students
5. Not related to math of prior grades
6. Not useful in math of later grades
7. I was taught it was inappropriate
8. Not emphasized in student text

	Was this procedure in the students' text?	1st Primary Reason	2nd Primary Reason	1st Primary Reason	2nd Primary Reason
41. Using arithmetical reasoning: Ex: Given $7x + 5 = 40$ What number increased by 5 is 40 (__ + 5 = 40)? Since the number is 35, then 7 times what number gives 35 (7 x __ = 35)? The solution is 5.	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐
42. Using trial and error: Ex: Given $7x + 5 = 40$ Try x = 4. But 7(4) + 5 = 33. So try x = 5, as x needs to be larger. 7(5) + 5 = 40. So, x = 5.	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐
43. Given $7x + 5 = 40$ Example Rules -- collect all constant terms on one side of the equation and all variable terms on the other. $7x = 40 - 5$ -- combine like terms. $7x = 35$ -- divide by the coefficient of x. $x = 5$	Yes ___1 No ___2				
___1 ___2 ___3		☐	☐	☐	☐

TEACHING TECHNIQUES

The following statements describe techniques a teacher might use in teaching formulas. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. *Emphasized (used as a primary technique, referred to extensively or frequently)*
2. *Used, but not emphasized*
3. *Not used*

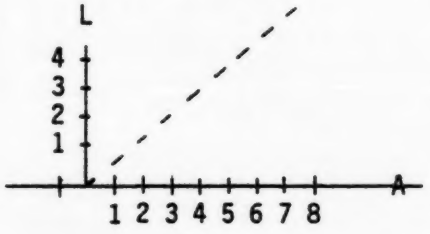
44. Presenting formulas and explaining the meaning of the terms in the formulas:

Ex: Formula: $A = \frac{1}{2} bh$
A stands for the area of a triangle
b stands for the base of a triangle
h stands for the height of a triangle

___1 ___2 ___3

45. Having the students inspect graphs and find formulas to express the relationships portrayed by the graph:

Ex:



A = 2 x L

___1 ___2 ___3

46. Providing data from which formulas or equations are developed:

x	y
0	0
1	3
2	5
3	7
4	9
5	11

Hence y = 2x + 1

___1 ___2 ___3

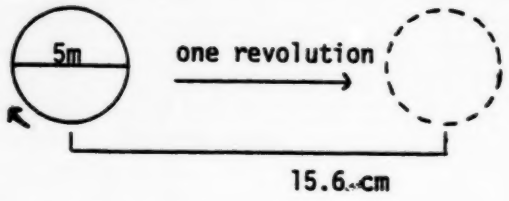
Teaching Techniques (Con't.)

RESPONSE CODES:

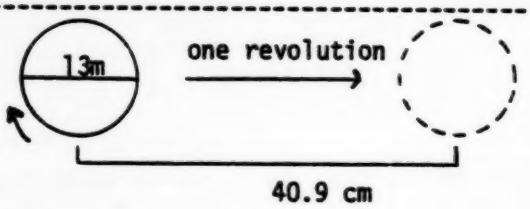
1. *Emphasized (used as a primary technique, referred to extensively or frequently)*
2. *Used, but not emphasized*
3. *Not used*

47. Having students collect data on related variables and formulate the relationship between the variables:

Ex.



Ratio: $\frac{15.6}{5} = 3.12$



Ratio: $\frac{40.9}{13} = 3.15$

Hence $\frac{c}{d} = 3.1$, So $C = 3.1d$ ____1 ____2 ____3

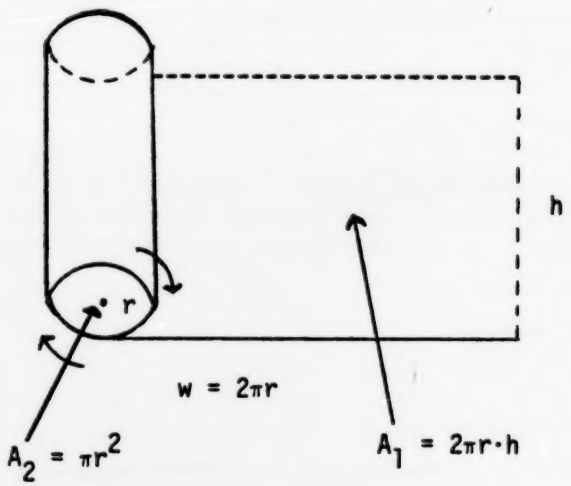
Teaching Techniques (Con't)

RESPONSE CODES:

1. *Emphasized (used as a primary technique, referred to extensively or frequently)*
2. *Used, but not emphasized*
3. *Not used*

48. Having students create new formulas based on known, simpler formulas:

Ex. Create formula for surface area of a cylinder based on formulas for area of the rectangle and the circle.



So, surface area = $2\pi rh + 2\pi r^2$

SA = $2\pi r\ (h + r)$

____ 1 ____ 2 ____ 3

APPLICATIONS AND PROBLEMS

Several types of problems are listed below which may have been included in your instructional program. CHECK the response code to indicate the degree to which a particular type of problem was studied by your class.

RESPONSE CODES:

1. *Emphasized (used as a primary type of problem, used extensively or frequently)*
2. *Used, but not emphasized*
3. *Not used*

49. Age problems

Roberta is now 15 years older than Stan. In 3 more years Roberta will be 3 times as old as Stan was 4 years ago. How old is Roberta now? ___1 ___2 ___3

50. Digit problems

If 4/5 of a number is added to 3/5 of that number, the result is the same as if 10 is added to the number. What is the number? ___1 ___2 ___3

51. Mixture problems

A feed dealer plans to mix corn (at $1.12 a bushel) with wheat (at $1.74 a bushel) to get a mixture that sells at $1.43 per bushel. How many bushels of corn are needed to make 200 bushels of the mixture? ___1 ___2 ___3

52. Percent problems

In 1980 about 4/7 of the telephones in Georgia had direct distance dialing capabilities. What percent was this? ___1 ___2 ___3

53. Distance-Rate-Time problems

How long does it take a rainstorm to travel 360 km at a rate of 45 km per hour? ___1 ___2 ___3

Applications and Problems (Con't)

RESPONSE CODES:

1. *Emphasized (used as a primary type of problem, used extensively or frequently)*
2. *Used, but not emphasized*
3. *Not used*

54. Interest problems

Les borrowed $3000 from the bank at 11% interest per year. How much interest would he have to pay at the end of 9 months? ___1 ___2 ___3

55. Area-Volume problems

The Great Pyramid in Eqypt has a square base measuring 240 m on a side. Its altitude is 160 m. What is its volume? ___1 ___2 ___3

56. Physical-Natural Science problems (lever problems, Hooke's law, ...)

If Sue has a mass of 56 kg and Sara has a mass of 42 kg, how far will Sue have to sit from the middle of the teeter-totter to balance with Sara, if Sara is 1.2 m from the middle? ___1 ___2 ___3

57. Energy or Ecological problems

An adult guppy requires 60 cm^2 of air surface to live in an aquarium. How many adult guppies can live in a rectangular aquarium that is 45 cm long and 30 cm wide? ___1 ___2 ___3

SOURCES OF APPLICATIONS AND PROBLEMS

Several sources of applications/problems of integers, formulas, and equations are listed below. CHECK the response code to show how frequently each source was used.

RESPONSE CODES:

1. *Used frequently*
2. *Used occasionally*
3. *Not used at all*

58.	Students' textbooks	___1	___2	___3
59.	Supplementary textbooks or workbooks	___1	___2	___3
60.	Worksheets or exercises designed by myself or local teachers	___1	___2	___3
61.	The curriculum guide or syllabus	___1	___2	___3
62.	Articles or papers published by professional associations	___1	___2	___3
63.	Applications or problems suggested by my students	___1	___2	___3
64.	Applications or problems from real world sources such as newspapers or individuals involved in the use of mathematics	___1	___2	___3

TIME ALLOCATIONS

65. What was the average length (in minutes) of each class period?..... ☐☐

INTEGERS

66. How many total class periods did you spend on the development of the integers and operations with integers? *(Combine partial periods when necessary)*..... ☐☐

Indicate the number of class periods spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

67. Activities related to the development of the concept of positive and negative integers..... ☐☐

68. Activities related to the addition of integers (positive and negative)..... ☐☐

69. Activities related to the subtraction of integers (positive and negative)..... ☐☐

70. Activities related to the multiplication of integers (positive and negative)..... ☐☐

71. Activities related to the division of integers (positive and negative)..... ☐☐

72. Activities related to the structural properties of the set of integers (commutativity, associativity, distributivity, etc.)..... ☐☐

73. Activities related to order relations with the set of integers..... ☐☐

74. Application/problem solving activities related to integers (textbook word problems, problems related to real world problems, recreational problems, challenging problems, etc. ..... ☐☐

NOTE: THE SUM OF THE PERIODS GIVEN FOR ITEMS 67 TO 74 SHOULD NOT EXCEED THE NUMBER GIVEN FOR ITEM 66.

Time Allocations (Con't.)

FORMULAS AND EQUATIONS

75. How many total class periods did you spend on teaching formulas and equations? *(Combine partial lessons when necessary.)*..... ☐☐

Indicate the number of class periods spent on each of the following activities (that is, demonstrations, explanations, students doing computational exercises, using manipulatives, etc.) with your class. Round your answer to the nearest whole number.

76. Activities related to evaluation of formulas (for given values of the variables)..... ☐☐

77. Activities related to deriving formulas or equations..... ☐☐

78. Application/problem solving activities related to use of formulas (textbook word problems, problems related to real world problems, recreational problems, challenging problems, etc.)..... ☐☐

79. Activities related to solving literal equations..... ☐☐

80. Activities related to solving linear equations..... ☐☐

81. Application/problem solving activities related to the use of equations (textbook word problems, problems related to real world problems, recreational problems, challenging problems, etc.)..... ☐☐

NOTE: THE SUM OF THE PERIODS GIVEN FOR ITEMS 76 TO 81 SHOULD NOT EXCEED THE NUMBER GIVEN FOR ITEM 75.

OPINIONS

Indicate the extent to which you agree or disagree with each of the following statements for your class. CIRCLE the choice which best describes your feelings.

82. The use of the number line adds a lot to the teaching of integers.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

83. It is very important to justify the rules for multiplying integers.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

84. A great deal of practice is required in order for students to acquire competence in performing operations with directed numbers.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

85. It is important for students to understand how integers obey general laws like the distributive law, the associative law, etc.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

86. Average students are usually not satisfied with knowing only the rules for performing operations with integers; they want to know why the rules work.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

87. Most students find it difficult to appreciate the significance of studying the structural properties (additive inverse, order relation, distributive law, etc.) of the set of integers.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

88. Most students cannot be expected to master the use of letters for unknowns quickly; they have to become accustomed to this usage slowly over a long period of time.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

Opinions (Con't.)

89. Linear equations whose solution is a fraction (like $5x - 2 = 1$) are generally more difficult for students to solve than linear equations whose solution is an integer (like $6x - 3 = 15$).

Strongly Disagree[1] Disagree[2] Undecided[3] Agree[4] Strongly Agree[5]

90. In solving equations, it is important that students be able to justify each step in their solution procedure.

Strongly Disagree[1] Disagree[2] Undecided[3] Agree[4] Strongly Agree[5]

91. Solving linear equations by trial and error helps students understand the meaning of a solution.

Strongly Disagree[1] Disagree[2] Undecided[3] Agree[4] Strongly Agree[5]

92. The notion "solution set" (those values of the unknown which make the relation true) aids the students' comprehension of linear equations.

Strongly Disagree[1] Disagree[2] Undecided[3] Agree[4] Strongly Agree[5]

93. Average students have difficulty in solving word problems involving linear equations.

Strongly Disagree[1] Disagree[2] Undecided[3] Agree[4] Strongly Agree[5]

94. Average students have difficulty in translating verbal and written sentences into mathematical sentences, and vice versa.

Strongly Disagree[1] Disagree[2] Undecided[3] Agree[4] Strongly Agree[5]

95. Average students have difficulty with applications involving linear equations.

Strongly Disagree[1] Disagree[2] Undecided[3] Agree[4] Strongly Agree[5]

Opinions (Con't.)

96. When solving problems, it is important for students to first identify the type of problem (age, digit, mixture, etc.) being solved.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

97. Solving equations requiring students to justify the steps in the solution procedure has a detrimental effect on learning how to solve equations.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

98. The notion of equivalent equations is useful in helping students understand solutions.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

99. Formulas taught should be memorized by students.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

100. Formulas should be used mainly to aid students in solving classes of story problems.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

101. Formulas should be used mainly to find volumes, areas, and perimeters of geometric figures.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

102. Formulas should be used mainly in applications to practical situations.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

APPENDIX E

SAMPLE QUESTIONNAIRE - GRADE 12

INTERNATIONAL ASSOCIATION for the
EVALUATION of EDUCATIONAL ACHIEVEMENT

SECOND
Study of
MATHEMATICS

GRADE 12
TOPIC SPECIFIC QUESTIONNAIRE
TRIGONOMETRY

(Booklet 27L)

FOR NATIONAL CENTRE USE ONLY

PROVINCE OF BRITISH COLUMBIA
MINISTRY OF EDUCATION
DIVISION OF PUBLIC INSTRUCTION
LEARNING ASSESSMENT BRANCH

This questionnaire deals with the content covered and the methods used in the teaching of trigonometry in the twelfth grade. You should complete this questionnaire if and only if your class has studied topics associated with trigonometry during the study year. If not, indicate so by checking box 1 and return the questionnaire to B.C. Research in the envelope provided. If you have covered topics dealing with trigonometry, check either box 2 or box 3 and continue with the remainder of the questionnaire.

1. ☐ 1 My class did not study any topics in trigonometry this year.

2. ☐ 2 Check here if your class has had the equivalent of a full semester course in trigonometry this year.

3. ☐ 3 Check here if your class has studied some of the topics associated with trigonometry this year, but has not had the equivalent of a full semester course in trigonometry.

4. Please list the title, author, publisher, and publication date of the main text you used with your class in the study of trigonometric topics. Please give the chapters you covered also.

   Title: _____

   Author: _____

   Publisher: _____ Date: _____

   Chapters Covered: _____

5. List additional texts by author and title.

   _____

   _____

Part I: Topics

The following topics are sometimes included in trigonometry. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. *taught for the first time*
2. *reviewed and then extended*
3. *reviewed only*
4. *assumed as prerequisite knowledge and neither taught nor reviewed*
5. *not taught and not assumed as prerequisite knowledge*

For each topic TAUGHT OR REVIEWED (i.e., if you checked response 1, 2 or 3 in the first column) write the numbers of the response codes which show the two primary reasons for including it in your course.

RESPONSE CODES:

1. *well known to me*
2. *in B.C. or local curriculum guide*
3. *easy for students to understand*
4. *enjoyed by students*
5. *related to math of prior grades*
6. *useful in math of later grades*
7. *I was taught it was appropriate*
8. *emphasized in student text*

For each topic NEITHER TAUGHT NOR REVIEWED (i.e., if you checked 4 or 5 in the first column) write the number of the response codes which show the two primary reasons for excluding it from your course.

RESPONSE CODES:

1. *not well known to me*
2. *not in B.C. or local curriculum guide*
3. *hard for students to understand*
4. *disliked by students*
5. *not related to math of prior grades*
6. *not useful in math of later grades*
7. *I was taught it was inappropriate*
8. *not emphasized in student text*

	1st Primary Reason	2nd Primary Reason	1st Primary Reason	2nd Primary Reason
6. Standard position for angles with vertex at the origin ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
7. Definition of the six trig functions from a right triangle ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
8. Definition of the trig functions from the unit circle ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
9. Definition of the trig functions using a wrapping function with the unit circle ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
10. Periodic properties of the trig functions ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
11. Graphs of the sine, cosine, and tangent functions ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
12. Graphs of the secant, cosecant, and cotangent functions ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
13. Graphs of general trig functions such as $y = A \sin (Bx + C)$ ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
14. Sum and difference of angles identities (reduction formulas) ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐

Part I: Topics

The following topics are sometimes included in trigonometry. CHECK the response code which describes the treatment of each topic in your class.

RESPONSE CODES:

1. *taught for the first time*
2. *reviewed and then extended*
3. *reviewed only*
4. *assumed as prerequisite knowledge and neither taught nor reviewed*
5. *not taught and not assumed as prerequisite knowledge*

For each topic TAUGHT OR REVIEWED (i.e., if you checked response 1, 2 or 3 in the first column) write the numbers of the response codes which show the two primary reasons for <u>including</u> it in your course.

RESPONSE CODES:

1. *well known to me*
2. *in B.C. or local curriculum guide*
3. *easy for students to understand*
4. *enjoyed by students*
5. *related to math of prior grades*
6. *useful in math of later grades*
7. *I was taught it was appropriate*
8. *emphasized in student text*

For each topic NEITHER TAUGHT NOR REVIEWED (i.e., if you checked 4 or 5 in the first column) write the number of the response codes which show the two primary reasons for <u>excluding</u> it from your course.

RESPONSE CODES:

1. *not well known to me*
2. *not in B.C. or local curriculum guide*
3. *hard for students to understand*
4. *disliked by students*
5. *not related to math of prior grades*
6. *not useful in math of later grades*
7. *I was taught it was inappropriate*
8. *not emphasized in student text*

	1st Primary Reason	2nd Primary Reason	1st Primary Reason	2nd Primary Reason
15. Double and half angle identities (reduction formulas) ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
16. Inverses of the sine, cosine, and tangent functions ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
17. Solution of trigonometric equations (e.g., 2 tan 3X = 2) ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
18. Solution of right triangles given sides or angles ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
19. Applications of the law of sines, cosines, or tangents ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
20. Graphs of Arcsin θ, Arccos θ, and Arctan θ ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
21. Polar coordinates ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
22. DeMoivre's Theorem ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
23. Graphing in polar coordinates ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐
24. Inverses of the secant, cosecant, and cotangent functions ___1 ___2 ___3 ___4 ___5	☐	☐	☐	☐

For each of the following topics indicate, as best you can, the number of full class periods you spent discussing the topic. Do not attempt to include the reviewing of the topics as they came up later in the year. Consider only the periods spent where the topic was the main focus of instruction.

	Topic	Number of Class Periods
25.	Standard positions for angles with vertex at origin	☐☐
26.	Definition of the six trig functions from a right triangle	☐☐
27.	Definition of trig functions from a unit circle	☐☐
28.	Definition of trig functions using a wrapping function	☐☐
29.	Periodic properties of the trig functions	☐☐
30.	Graphs of the sin, cos, and tan functions	☐☐
31.	Graphs of the sec, csc, and cot functions	☐☐
32.	Graphs of the general trig function $y = A \sin(Bx + C)$	☐☐
33.	Sum and difference of angle identities	☐☐
34.	Double and half angle identities	☐☐
35.	Inverses of the sin, cos, and tan functions	☐☐
36.	Solution of trig equations ($2 \tan 3x = 2$)	☐☐
37.	Solution of right triangles	☐☐
38.	Applications of law of sines, cosines, tangents	☐☐
39.	Graphs of Arcsin θ, Arccos θ, Arctan θ.	☐☐
40.	Polar coordinates	☐☐
41.	DeMoivre's Theorem	☐☐
42.	Graphing in polar coordinates	☐☐
43.	Inverses of the sec, csc, and cot functions	☐☐
44.	Graphing Arcsec θ, Arccsc θ, Arccot θ.	☐☐

Several formulas (theorems) involving the trigonometric functions can be developed during the study of trigonometry. For each of the following formulas CHECK the one response which best describes the manner in which you initially presented the formula to your class.

RESPONSE CODES:

1. I gave a formal proof for the formula.
2. I stated the formula and gave an informal justification (graphic or verbal).
3. I stated the formula and used it without any justification.
4. I did not cover the formula or discuss it with my class.

If you gave a formal proof for the formula, (i.e. if you checked response 1 in the first column) CHECK the one response code below which indicates your PRIMARY reason.

RESPONSE CODES:

1. textbook gave proof
2. proof unifies basic ideas
3. interesting example of mathematical reasoning

CHECK the one response code below which best describes your expectation of student mastery for this formula (theorem). (Omit if you did not cover the formula.)

RESPONSE CODES:

1. prove and apply formula
2. informal justification and apply
3. recall and apply formula
4. apply formula when given

---

45. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

46. $\cos 2\theta = 2\cos^2\theta - 1$

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

47. $\sin 2\theta = 2\sin\theta\cos\theta$

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

48. $\tan 2\theta = (2\tan\theta)/(1 - \tan^2\theta)$

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

49. $\cos\frac{\theta}{2} = \pm\sqrt{(1+\cos\theta)/2}$

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

50. $\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

51. $\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta) + \sin(\alpha+\beta)\right]$

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

52. $\tan(\alpha+\beta) = (\tan\alpha+\beta)/(1 - \tan\alpha\tan\beta)$

___1 ___2 ___3 ___4 ___1 ___2 ___3 ___1 ___2 ___3 ___4

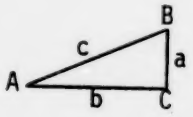
PART II: Specific Methods and Content Organization

Three interpretations of the basic trigonometric functions (sine, cosine, tangent) are illustrated in the following three items. CHECK the response code which describes the treatment of each interpretation in your class.

RESPONSE CODES:

1. *Emphasized (used as a primary explanation, referred to extensively or frequently).*
2. *Used, but not emphasized.*
3. *Not used.*

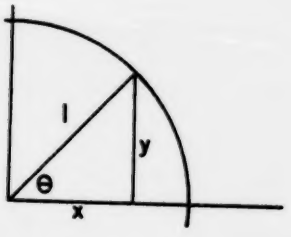
53. Ratio Interpretation: The three basic trigonometric functions are defined and illustrated as ratios of the lengths of sides of a right triangle.



sin A = a/c
cos A = b/c
tan A = a/b

____1 ____2 ____3

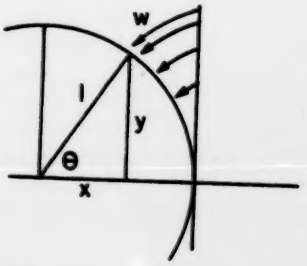
54. Unit Reference Circle: The three functions are defined and illustrated as in the ratio interpretation except the triangle has hypotenuse 1 and point B is located on a unit radius circle with centre at the origin of the coordinate plane. Here the functions map the real angle measures into real numbers.



____1 ____2 ____3

$\sin\theta = y$, $\cos\theta = x$, $\tan\theta = y/x$

55. Wrapping Function: First a wrapping function W is defined that maps the real number line onto the circumference of the unit circle. The trigonometric functions are then defined so that they map the real number angle measures (modulo 2π) to the real numbers.



____1 ____2 ____3

$\sin\theta = y$, $\cos\theta = x$, $\tan\theta = y/x$

56. If you used the ratio interpretation for the development of the trigonometric functions, why did the original use of the triangle model appear? (*CHECK one.*)

In order to motivate the definitions of the functions. ____1

In an attempt to consider applications of the trig functions. ____2

I did not use the ratio interpretation with the target class. ____3

Many of the students in Grade 12 mathematics enter the course with some understanding of the trigonometric functions. Relative to the three interpretations given in items 53 to 55 above, CHECK the level of understanding of each interpretation your students brought to the class.

RESPONSE CODES:

1. *They had a thorough understanding of the interpretation.*
2. *They had a minimal understanding of the interpretation.*
3. *They were unaware of the interpretation.*

57. Ratio Interpretation . . . . . ___1 ___2 ___3

58. Unit Circle Interpretation . . . . . ___1 ___2 ___3

59. Wrapping Function Interpretation . . . . . ___1 ___2 ___3

For each of the three interpretations given in items 53 to 55, give your impression of the level of understanding of the average student in your class at the end of your study of trigonometric functions.

RESPONSE CODES:

1. *The average student had a thorough understanding of the interpretation.*
2. *The average student had a minimal understanding of the interpretation.*
3. *The average student was unaware of the interpretation.*

60. Ratio Interpretation . . . . . ___1 ___2 ___3

61. Unit Circle Interpretation . . . . . ___1 ___2 ___3

62. Wrapping Function Interpretation . . . . . ___1 ___2 ___3

For each of the interpretations of the trigonometrics functions for which you provided instruction, WRITE the numbers of the response codes showing the two primary reasons you decided to discuss that interpretation.

RESPONSE CODES:

1. *I knew this interpretation quite well.*
2. *This interpretation is emphasized in the B.C. or a local curriculum guide.*
3. *This interpretation is easy for students to understand.*
4. *This interpretation is enjoyed by the students.*
5. *This interpretation relates closely to math studied in prior grades.*
6. *This interpretation is useful in the study of math in subsequent grades.*
7. *I was taught that this was the appropriate interpretation.*
8. *This interpretation was presented by the textbook.*

	1st Primary Reason	2nd Primary Reason
63. Ratio Interpretation	☐	☐
64. Unit Circle Interpretation	☐	☐
65. Wrapping Function Interpretation	☐	☐

For each of the interpretations of the trigonometric functions which was <u>not</u> emphasized in your class, write the numbers of the response codes showing the two primary reasons why you did <u>not</u> emphasize the interpretation.

RESPONSE CODES:

1. *I never considered using this interpretation.*
2. *This interpretation is not emphasized nor recommended by the B.C. or a local curriculum guide.*
3. *This interpretation is difficult for my students to understand.*
4. *This interpretation is disliked by the students.*
5. *This interpretation requires background knowledge the students do not have.*
6. *This interpretation is not useful in the further study of mathematics.*
7. *I was taught this interpretation was inappropriate.*
8. *This interpretation was not in the textbook.*

	1st Primary Reason	2nd Primary Reason
66. Ratio Interpretation	☐	☐
67. Unit Circle Interpretation	☐	☐
68. Wrapping Function Interpretation	☐	☐

69. In graphing trigonometric functions, I covered and expected student mastery of graphing of: (*CHECK all that apply*).

the various functions with varied periods through a full cycle . . . . . ___1

trigonometric functions of the form $y = A \cos (Bx + C)$ through a full cycle . . . . . ___1

I only covered the graphs of the standard functions with their natural periods and amplitudes . . . . . ___1

I did not do any graphing of trigonometric functions . . . . . ___1

70. In graphing trigonometric functions, I showed the class how to: (*CHECK all that apply*).

use a hand calculator to determine functional values . . . . . ___1

use topics from the algebra of functions including products and composition . . . . . ___1

stress the role of the constant terms in the general trigonometric equation $y = a \cos (bx + c)$ in determining the shape of the graph . . . . . ___1

I did not teach graphing of trig functions . . . . . ___1

71. In solving problems involving inverse trigonometric functions, I taught the class to: (Use as an example the problem $\sin \{\text{Arccos } (3/5)\}$.)

solve the problem by asking for the sine of the angle whose cosine is 3/5 and then using the algebraic approach of substituting 3/5 into an identity and solving for the sine of the required angle . . . . . ___1

solve the problem by drawing a sketch of the related triangle with an angle of θ having a side adjacent of 3 and a hypotenuse of 5. Using the Pythagorean Theorem find the opposite side of 4 and then find the $\sin \theta$ . . . . . ___1

solve the problem using both of the above methods . . . . . ___1

72. When confronted with a problem such as $\sin (\text{Arcsin } x)$ I showed the class to employ $f(f^{-1}(x)) = x$ to get the solution.

Yes . . . . . ___1

No. . . . . ___2

73. I developed the Law of Cosines using:

the unit circle and the distance formula . . . . . ___1

an arbitrary triangle and the distance formula . . . . . ___1

using vector methods . . . . . ___1

I did not give a proof of the Law of Cosines, but we used it. . . . . ___1

using an arbitary triangle, the Pythagorean Theorem, and trig definitions . . . . . ___1

my class did not discuss the Law of Cosines . . . . . ___1

74. In working with standard angles, I developed the values for 0, 30, 45, 60 and 90 degree angles using:

ratios from elementary geometry. . . . . ___1

the unit circle and graph paper. . . . . ___1

the unit circle and the results of elementary geometry plus the Pythagorean Theorem. . . . . ___1

hand calculators . . . . . ___1

I just stated the values and we took them as given . . . . . ___1

We did not use the values of these standard angles . . . . . ___1

75. In dealing with applications of trigonometric functions I assigned problems for the students in the target class involving: (*CHECK all that apply*)

surveying. . . . . ___1

electrical work. . . . . ___1

inaccessible distances . . . . . ___1

periodic motion. . . . . ___1

friction . . . . . ___1

vector applications. . . . . ___1

76. In getting application problems for the class, I depended upon: (*CHECK all that apply*)

problems from the textbook	___1
supplementary textbooks	___1
problems I created	___1
articles in professional journals	___1
problems I got at a provincial or NCTM meeting	___1
the curriculum guide	___1
I used no application problems	___1

77. For calculations involving trigonometric function values, my students: (*CHECK all that apply*)

used trig tables	___1
used log and trig tables	___1
used calculators and trig tables	___1
used calculators having the trig functions in them	___1
used computers	___1

78. What methods of evaluation did you use in marking your students' work in trigonometry? (*CHECK all that apply*)

grading of daily paper work	___1
grading of student board work	___1
daily quizzes	___1
surprise quizzes	___1
chapter tests	___1
projects	___1

79. Did you use any manipulative models in the teaching of trigonometry?

No . . . . . ___1

Yes (*Please list*). . . . . ___1

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

_____

PART III: Time Allocations and Instructional Emphasis

80. How many class periods of mathematics did your class spend on the study of trigonometry? ☐☐ periods

81. What was the average length of each of these periods? ☐☐ minutes

82. How many of the class periods were spent on the study of trigonometric identities? ☐☐ periods

83. How many of the class periods were spent on the applications of trigonometric functions? ☐☐ periods

84. How many of the periods were spent on the study of graphing of trigonometric functions? ☐☐ periods

Estimate the percent of an average instructional period spent on each of the following teaching/learning activities. Make sure the percents add to 100.

85. The percent of time devoted to teacher lecture was about ☐☐ %

86. The percent of time devoted to discussion of homework by the teacher was about ☐☐ %

87. The percent of time devoted to discussion of homework by the students was about ☐☐ %

88. The percent of time devoted to students studying in class in small working groups was about ☐☐ %

89. The percent of time devoted to student studying individually in class was about ☐☐ %

Indicate the level of emphasis placed on the following topics and levels of behaviour by checking the response code according to the following levels of emphasis:

RESPONSE CODES:

1. *Primary importance in the class*
2. *Secondary importance in the class*
3. *Little importance in the class*
4. *No importance in the class*
5. *Students already had this mastery*

90. Skill in solving for missing parts of right triangles.

_____1 _____2 _____3 _____4 _____5

91. Skill in proving trigonometric identities.

_____1 _____2 _____3 _____4 _____5

92. Skill in solving equations involving trigonometric functions.

_____1 _____2 _____3 _____4 _____5

93. Developing trigonometric function curve-sketching skills.

_____1 _____2 _____3 _____4 _____5

94. Developing algebraic techniques for dealing with trig problems.

_____1 _____2 _____3 _____4 _____5

95. Applying trigonometric skills to solve application problems.

_____1 _____2 _____3 _____4 _____5

PART IV: Opinions

Indicate by CIRCLING the extent to which you agree or disagree with each of the following statements concerning the teaching of trigonometry and topics associated with it.

96. Students should learn the basic identities through the double angle and half angle forms and be able to recall them at will.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

97. Students should be allowed to use a hand calculator with trig functions at all levels of trigonometry study and testing.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

98. The study of trigonometry should involve a large number of applications.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

99. Students should be expected to solve trig problems with paper, pencil, and a set of trig tables only.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

100. A large portion of the time in trigonometry should be spent in the study of the functional aspects of trigonometry, i.e., periodicity, amplitude, inverse functions, etc.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

101. Students should be expected to graph the basic functions and the general trigonometric functions.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

102. Students should be expected to graph the inverse trigonometric functions and relate their graphs to the graphs of the basic trigonometric functions.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

103. The role of trigonometry in the curriculum is overemphasized.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

104. Schools should offer a semester course in trigonometry as opposed to a collection of trigonometric topics within the advanced algebra course or the senior level mathematics course.

Strongly Disagree[1]	Disagree[2]	Undecided[3]	Agree[4]	Strongly Agree[5]

105. The ratio-right triangle approach to trigonometric functions is to be preferred over the unit circle or wrapping function approach because of its historical roots.

Strongly Disagree[1]	Disagree[2]	Undecided [3]	Agree[4]	Strongly Agree[5]

APPENDIX F

ITEMS ON GRADE 8 ROTATED FORMS

A4.

The Davis family took a car trip from Anabru through Bergen to Chase. They then drove back to Bergen through Carlville, and then returned to their home in Anabru. If the total distance they drove was 115 kilometres, how far is it from Anabru to Bergen?

A 20 kilometres

B 35 kilometres

23.9 C 40 kilometres

D 75 kilometres

E 80 kilometres

A5. A number x is multiplied by itself and the result is added to four times the original number. This can be expressed as:

A $x^2 + 4$

B $x + 4$

C $2x + 4$

D $x(x^2 + 4)$

55.3 E $x^2 + 4x$

A26. If $x + 3$, the value of $-3x$ is

A -9

B -6

C -1

D 1

50.9 E 9

A28. When $x = 2$, $\frac{7x + 4}{5x - 4}$ is equal to

A 11

59.5 B 3

C $\frac{11}{5}$

D $\frac{9}{5}$

E $\frac{7}{5}$

A33. The table below compares the height from which a ball is dropped (d) and the height to which it bounces (b).

d	50	80	100	150
b	25	40	50	75

Which formula describes this relation?

A $b = d^2$

B $b = 2d$

34.7 C $b = \frac{d}{2}$

D $b = d + 25$

E $b = d - 25$

A35.

Which of the following sequences of numbers is in the order in which they occur from left to right on the number line?

A $0, \frac{1}{2}, -1$

B $0, -1, \frac{1}{2}$

49.6 C $-1, -\frac{1}{2}, 0$

D $-1, 0, -\frac{1}{2}$

E $-\frac{1}{2}, -1, 0$

B5.

m	-1	1	2	4
n	-1	3	5	9

For the table shown, a formula that could relate m and n is

A $n = m$

B $n = 3m$

C $n = -m^2 + 1$

D $n = m^2 + 1$

27.9 E $n = 2n + 1$

B12. If $6x - 3 = 15$
then $6x = 15 - 3$ (i)
and $6x = 12$ (ii)
and $x = \frac{12}{6}$ (iii)
and $x = 2$ (iv)

The error in the above reasoning if one exists, FIRST APPEARS in line

43.6 A (i)

B (ii)

C (iii)

D (iv)

E None of these, there is no error

B22. Find the value of N.

$N = 10^1 + 10^1 + 10^0 + 10^{-2}$

A $N = 0$

B $N = 20$

31.4 C $N = 1011.01$

D $N = 100$

E Some other value

B24. Simplify: $5x + 3y + 2x - 4y$

A $7x + 7y$

B $6x - 2y$

C $6xy$

33.6 D $7x - y$

E $7x + y$

B29. If y dollars are shared equally among four boys, how many dollars does each boy receive?

A $y - 4$

B $\frac{4}{y}$

C 4

56.6 D $\frac{y}{4}$

E $4y$

B30. $(-6) - (-8)$ is equal to

A 14

48.2 B 2

C -2

D -10

E -14

Algebra items - Forms A and B

C12. The cost of printing greeting cards consists of a fixed charge of 100 cents and a charge of 6 cents for each card printed. Which of the following equations can be used to determine the cost of printing n cards?

53.4 A cost = $(100 + 6n)$ cents

B cost = $(106 + n)$ cents

C cost = $(6 + 100n)$ cents

D cost = $(106n)$ cents

E cost = $(600n)$ cents

C13. "Six times a certain number (call it q) equals the sum of eight and twice the number". This can be written as

A $6q + 2(8 + q)$

B $6(q + 8) + 2q$

C $6(q + 8) = 8 + 2q$

62.2 D $6q + 8 + 2q$

E $q + 1$

C16. If $x + y = a - 1$, then $\frac{x - a}{x + y}$ is equal to

A -2

B -1

41.8 C 0

D $\frac{1}{2}$

E 1

C24. Which of the following is FALSE when a, b, and a are different real numbers?

A $(a + b) + a = a + (b + a)$

B $ab = ba$

C $a + b = b + a$

D $(ab)a = a(ba)$

44.0 E $a - b = b - a$

C26. A bowling ball travels 4 metres per second. The distance in metres travelled in s seconds is given by $d=4s$. In the tab e below x is equal to

s	d
0	0
1	4
2	8
3	x
4	16

A 6

B 10

75.7 C 12

D 14

E None of these

C35. $\frac{a}{15} - \frac{b}{5}$ is equal to

27.0 A $\frac{a-3b}{15}$

B $\frac{5a-15b}{15}$

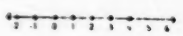
C $\frac{a-b}{10}$

D $\frac{a-b}{75}$

E None of these

D5. The set of integers less than 5 is represented on one of the number lines shown below. Which one?

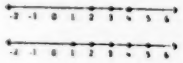
63.7 A



B

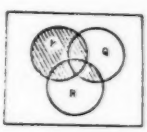
C

D



E

D17. The symbol $P \cap Q$ represents the intersection of sets P and Q and the symbol $P \cup Q$ represents the union of sets P and Q. Which of the following represents the shaded portion of the diagram below?



A $(P \cap Q) \cup R$

22.2 B $P \cup (Q \cap R)$

C $P \cap (Q \cup R)$

D $(P \cap Q) \cap R$

E $(P \cup Q) \cap R$

D26. $\frac{x}{2} < 7$ is equivalent to

A $x < \frac{7}{2}$

B $x < 5$

37.4 C $x < 14$

D $x > 5$

E $x > 14$

D27. Soda costs a cents for each bottle, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?

A $ax + by$ cents

33.8 B $ax - by$ cents

C $(a - b)x$ cents

D $(a + x) - (b + y)$ cents

E None of these

D31. $-5(6 - 4)$ is equal to

A 50

B 26

C 10

74.1 D -10

E -26

D20. Which equation is true for ALL values of n?

78.4 A $2 + n = n + 2$

B $3 + n = 4 + 2$

C $n + 1 = 1$

D $2n + 1 = n$

E $n + 3 = 3n$

Algebra items - Forms C and D

A1. Which of the following is equal to a quarter of a million?

A 25,250

B 40,000

C $\frac{1}{4}$,000,000

77.8 D 250,000

E 2,500,000

A17 1st row 1
2nd row 1 - 1
3rd row 1 - 1 + 1
4th row 1 - 1 + 1 - 1
5th row 1 - 1 + 1 - 1 + 1

What is the sum of the 50th row?

37.7 A 0

B 1

C 2

D 25

E 30

A24. If $10^4 \times 10^3 = 10^n$ then n is equal to

A 4

77.3 B 5

C 6

D 8

E 9

A30. What is the square root of 12 x 75?

A 6.25

49.8 B 30

C 87

D 625

E 900

A32. Peter and Paul decided to start saving money. Peter can save 3 dollars each month and Paul can save 5 dollars. At this rate, after how many months will Paul have exactly 10 dollars more than Peter?

A 2

B 3

C 4

63.6 D 5

E 8

B15. Which of the following operations with whole numbers will ALWAYS give a whole number?

I Addition
II Multiplication
III Division

A I only

B II only

C III only

55.7 D I and II only

E II and III only

B16. The value of $2^3 \times 3^2$ is

A 30

B 36

C 64

67.1 D 72

E None of these

B20. Which of these numbers is a prime number?

A 21

B 22

78.3 C 23

D 24

E 25

B33. $\sqrt{75}$ is between

A 4 and 5

B 5 and 6

C 6 and 7

D 7 and 8

57.2 E 8 and 9

B34. $(22 \times 18) - (47 + 59)$ is equal to

70.2 A 290

B 300

C 384

D 408

E 502

C 8. A group of children was divided into 7 teams with nine in each team. Later, the same group of children was divided into teams with seven in each team. How many teams were there then?

A 7

B 8

79.4 C 9

D 16

E 63

C 19. One bell rings every 8 minutes, a second bell rings every 12 minutes. The both ring at exactly 12 o'clock. In how many minutes will they next ring together?

A 8

B 12

C 20

41.9 D 24

E 96

C 21. 3.23×10^5 is equal to

A 0.0000323

B 3.23000

C 32,300

46.4 D 323,000

E 32,300,000

C 28. 162 x 45 is equal to

A 1378

B 1458

C 5890

D 6290

83.3 E 7290

C 33. Since $4 \times 9 = 36$, $\sqrt{36}$ is equal to

A 4 x 9

B 4 x 3

C 2 x 9

32.8 D 2 x 3

E $\sqrt{4} \times \sqrt{9}$

D 6. Which of the following is (are) TRUE?

I $(53 \times 73) \times 17 = 53 \times (73 \times 17)$

II $133 \times (78+89) = (133 \times 78) + 89$

III $133 \times (78+89) = (133 \times 78) + (133 \times 89)$

A I only

B II only

C III only

D I and II only

23.9 E I and III only

D 10. Find the sum:

3 weeks 5 days
+ 9 weeks 6 days

A 12 weeks 1 day

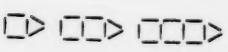
B 12 weeks 4 days

C 13 weeks 1 day

70.5 D 13 weeks 4 days

E 13 weeks 11 days

D 12. Matchsticks are arranged as follows:



If the pattern is continued, how many matchsticks are used in making the 10th figure?

A 30

50.6 B 33

C 36

D 39

E 42

D 13. 1054
-865

86.1 A 189

B 199

C 211

D 289

E 299

D 28. Which of the following equals $7 \times (3 + 9)$?

59.8 A $(7 \times 3) + (7 \times 9)$

B $(7 \times 9) + (3 \times 9)$

C $(7 \times 3) + (3 \times 9)$

D 7×27

E $21 + 9$

Arithmetic items - Forms C and D

A3. Which is the closest estimate for the answer to $5\frac{3}{7} \times 6\frac{5}{8}$?

A about 8
B about 11
73.7 C about 12
D about 15
E about 31

A13. $\frac{3}{8} - \frac{1}{5}$ is equal to

A $\frac{1}{20}$
67.4 B $\frac{7}{40}$
C $\frac{7}{20}$
D $\frac{19}{40}$
E $\frac{2}{3}$

A18.

The position on the scale indicated by the arrow is

A 1.004
64.4 B 1.04
C 1.08
D 1.4
E 1.8

A21. $.004\overline{)24.56}$

In the division above, the correct answer is

A 0.614
B 6.14
C 61.4
D 614
52.8 E 6140

B2. Which of the following is thirty-seven thousandths?

A 37000
B 37
C 0.37
41.4 D .037
E 0.0037

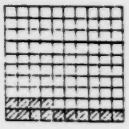
B7. Alexandra walked from Riverview to Bridgeport, which are 3.1 kilometres apart. During her walk she lost her watch, went back 1.7 kilometres to find it, and then continued in the original direction until she reached Bridgeport. How many kilometres had Alexandra walked altogether when she arrived at Bridgeport?

A 1.4
B 4.8
44.5 C 6.5
D 8.7
E None of these

B10. $\frac{\frac{3}{5}}{\frac{2}{7}}$ is equal to

34.0 A $\frac{21}{10}$
B $\frac{5}{12}$
C $\frac{10}{21}$
D $\frac{6}{35}$
E $\frac{31}{35}$

B27.



The large square has area 1 square unit. The area of the shaded part is

A 14 square units
B 1.4 square units
53.3 C 0.14 square units
D 0.014 square units
E 0.0014 square units

B35. There are 35 students in a class. $\frac{1}{5}$ of them come to school by bus, another $\frac{2}{5}$ come by bicycle. How many come to school by other means?

A 7
59.0 B 14
C 21
D 28
E 35

Fractions items - Froms A and B

C3. Which of the following is a pair of equivalent fractions?

A $\frac{5}{0}$ and $\frac{2}{3}$

B $\frac{5}{6}$ and $\frac{2}{3}$

C $\frac{4}{5}$ and $\frac{14}{15}$

87.8 D $\frac{3}{5}$ and $\frac{9}{15}$

E $\frac{1}{2}$ and $\frac{14}{15}$

C5. $1\frac{2}{5} - \frac{1}{2}$ is equal to

A $\frac{2}{3}$

64.4 B $\frac{9}{10}$

C $1\frac{1}{10}$

D $1\frac{1}{7}$

E $1\frac{1}{3}$

C22. The speed of sound is approximately 340 metres per second. How long will it take before the sound of a car horn reaches your ears if the car is 714 metres away?

A 0.21 seconds

60.5 B 2.1 seconds

C 21 seconds

D 210 seconds

E None of the

C25. 74.236 rounded to the nearest hundredth is

A 74.2

B 74.3

C 74.23

56.9 D 74.24

E 74.240

D1. The picture shows some black and some white marbles. Of all these marbles what fraction is white?

A $\frac{1}{2}$

B $\frac{6}{4}$

C $\frac{4}{6}$

52.8 D $\frac{6}{10}$

E $\frac{4}{10}$

D3. A runner ran 3,000 metres in exactly 8 minutes. What was his average speed in metres per second?

A 3.75

25.6 B 6.25

C 16.0

D 37.5

E 62.5

D25. $\frac{1}{2} \times \frac{1}{4}$ is equal to

73.1 A $\frac{1}{8}$

B $\frac{1}{6}$

C $\frac{2}{8}$

D $\frac{2}{4}$

E 8

D33.

847.36

In the number in the box the digit 6 represents

70.3 A $6 \times \frac{1}{100}$

B $6 \times \frac{1}{10}$

C 6×1

D 6×10

E 6×100

Fractions items - Forms C and D

A2. In which diagram below is the second figure the image of the first figure under a reflection in a line?

A

B

74.8 C

D

E

A4.

The triangles shown above are congruent. The measures of some of the sides and angles are as shown. What is x?

A 52

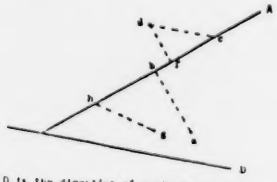
30.7 B 55

C 65

D 73

E 75

A8.



If D is the direction of projection and A is the axis of projection, which of the following statements is correct?

A p(a) = b

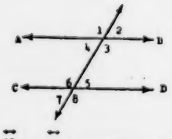
14.8 B p(d) = c

C p(d) = f

D p(g) = h

E p(c) = d

A10.



Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are parallel. Two angles whose measures must add up to 180° are:

A ∠1 and ∠3

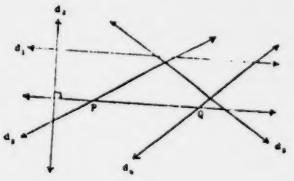
32.6 B ∠4 and ∠6

C ∠2 and ∠5

D ∠2 and ∠7

E ∠1 and ∠8

A14.



Among the following lines d_1, d_2, d_3, d_4, d_5, which has no point equidistant from P and from Q?

A d_1

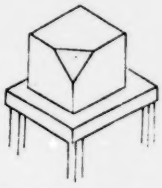
20.5 B d_2

C d_3

D d_4

E d_5

A15.



The figure above shows a wooden cube with one corner cut off and shaded. Which of the following drawings shows how this cube would look when viewed from directly above it?

A

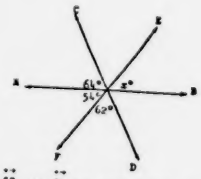
B

C

D

74.4 E

A27.



$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ are intersecting straight lines as shown above. The measures of certain angles are shown. x is equal to

54.5 A 54

B 62

C 64

D 126

E 128

A34. One of the following points can be joined to the point (-3,4) by a line segment which cuts NEITHER the x NOR the y axis. Which one?

23.1 A (-2,3)

B (2,-3)

C (2,3)

D (-2,-3)

E (4,-3)

B4.

The line m is a line of symmetry for figure ABCDE. The measure of angle BCD is

A 30°

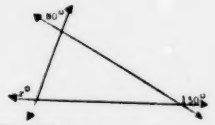
B 50°

58.3 C 60°

D 70°

E 110°

B8.



Three straight lines intersect as shown in the diagram. What is x equal to?

A 30

B 50

C 60

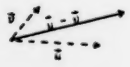
29.0 D 110

E 150

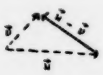
B11.

$\vec{u}$ and $\vec{v}$ are two vectors. Which figure represents $\vec{u} - \vec{v}$?

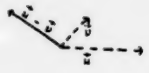
A



B

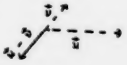


C

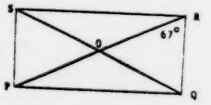


13.2 D

E



B13.



In the above rectangle the measure of $\angle ROQ$ is

A 23°

B 45°

28.5 C 46°

D 54°

E 67°

B14. $\triangle ABC$ and $\triangle A'B'C'$ are congruent and their corresponding sides are parallel. $\triangle ABC$ maps onto $\triangle A'B'C'$ by a

A reflection

B glide reflection

C rotation

D enlargement

14.7 E translation

B21.

AB ∥ DC and AD ∥ BC. Quadrilateral ABCD is a

A rhombus

55.1 B parallelogram

C square

D rectangle

E none of the above

B25.

Which of these is a correct statement for this triangle?

42.3 A $x^2 = 3^2 + 4^2$

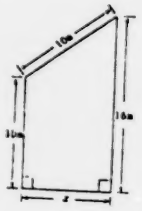
B $x^2 + 3^2 = 4^2$

C $x = 4^2 - 3^2$

D $x^2 = 4^2 - 3^2$

E $x = 4 + 3$

B26.



x is equal to

A 4 m

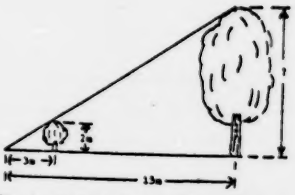
B 6 m

49.3 C 8 m

D 10 m

E 12 m

B32.



The picture above shows how Pedro used a short tree to find the height of the tall tree. What answer should Pedro get?

51.3 A 10 metres

B 12 metres

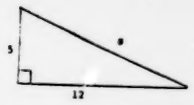
C 14 metres

D 17 metres

E 20 metres

Geometry items - Form B

C1.



What is the value of a?

A 7

33.5 B 13

C 15

D 17

E None of these

C4. Which of the following patterns can be folded along the dotted sides to make a cube?

A

B

59.6 C

D

E

C9. If two triangles are SIMILAR, which of the following statements is TRUE?

36.3 A Their corresponding angles MUST be congruent.

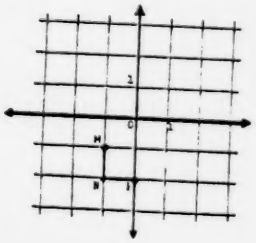
B Their corresponding sides MUST be congruent.

C Their corresponding sides MUST be parallel

D They MUST have the same area

E They MUST have the same shape and size.

C11.



Suppose you start at point M(-1,-1), move a distance of one unit to N(-1,-2), then turn left and move one unit to the point P(0,-2). If you again turn left and move one unit, you will now be at the point with coordinates

A (1, -2)

B (0, -3)

33.7 C (0, -1)

D (-1, -2)

E None of the above

C23 A quadrilateral MUST be a parallelogram if it has

A one pair of adjacent sides equal

B one pair of parallel sides

C a diagonal as axis of symmetry

D two adjacent angles equal

41.2 E two pairs of parallel sides

C29. If segment PQ were drawn for each figure shown below, it would divide one of the figures into two congruent triangles. Which figure?

A

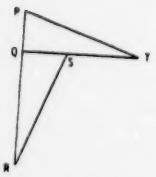
B

C

54.5 D

E

C32.



ΔPQT can be rotated (turned) onto ΔSQR. The centre of rotation is

A point P

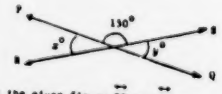
71.2 B point Q

C point R

D point S

E point T

C34.



If, in the given figure $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are intersecting straight lines, then $x + y$ is equal to

A 15

B 30

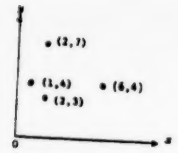
34.1 C 60

D 180

E 300

Geometry items - Form C

D4.



The straight line joining the points (2,3) and (2,7) cuts the straight line joining the points (1,4) and (6,4) at the point

A (4,2)

B (1,4)

C (1,3)

D (2,3)

43.0 E (2,4)

D15. On the level ground, a boy 5 units tall casts a shadow 3 units long. At the same time a nearby telephone pole 45 units high casts a shadow the length of which, in the same units, is

A 24

53.2 B 27

C 30

D 60

E 75

D16.

One of the following figures is congruent to the figure above. Which one?

A

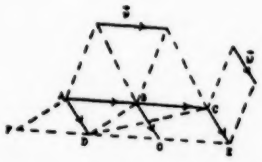
B

C

D

76.5 E

D18.



Given $\vec{u}$ and $\vec{w}$ as shown in the figure above, what is $\vec{DB}$, the vector from D to B

A $\vec{u} + \vec{w}$

13.5 B $\vec{u} - \vec{w}$

C $\vec{w} - \vec{u}$

D $-\vec{u} - \vec{w}$

E $\vec{u} + 2\vec{w}$

D19.

If AB is a straight line, what is the measure in degrees of angle BCD?

A 20

B 40

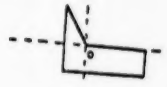
C 50

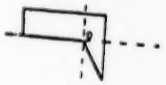
65.4 D 80

E 100

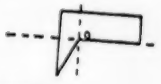
D 22.

A half-turn about O is applied to the figure above. Which of the figures below is the result?

A 

B 

C

36.3 D 

E

D 24. In a quadrilateral, two of the angles each have measure of 110°, and the measure of a third angle is 90°. What is the measure of the remaining angle?

23.1 A 50°

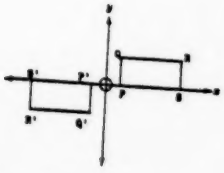
B 90°

C 130°

D 140°

E None of the above

D29.



PQRS is a rectangle. Its image after a transformation is the rectangle P'Q'R'S', as shown above. The transformation used could have been

28.2 A a rotation about the origin.

B a reflection in the y-axis

C a translation parallel to the x-axis

D a reflection in the x-axis

E a translation parallel to the y-axis.

Geometry items - Form D

A7. A 15 centimetre piece is cut from a ribbon 1 meter long. What is the length of the remaining piece?

85.2 A 85 cm

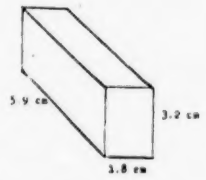
B 115 cm

C 985 cm

D 1015 cm

E 9985 cm

A9.



The figure above shows a rectangular box. Which of the following is closest to the volume of this box?

A 16 cm³

B 18 cm³

C 28 cm³

42.4 D 36 cm³

E 48 cm³

A20.

What is the area of the above parallelogram?

A 30 cm²

B 36 cm²

C 48 cm²

36.0 D 60 cm²

E 80 cm²

A29.

What is the area of triangle PQR?

A 3 square units

B 6 square units

38.8 C 9 square units

D 12 square units

E 18 square units

A31.

The figure QRST is a square and PQT an equilateral triangle. If PQ = 6 cm then the area of the square is

A 64 cm²

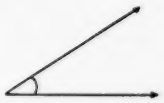
B 48 cm²

46.6 C 40 cm²

D 36 cm²

E 24 cm²

B17.



The measure of the angle shown is nearest to

A 155°

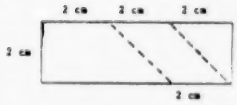
B 145°

C 50°

41.7 D 35°

E 15°

B18.



The rectangle shown above is cut along the dotted lines and the three parts put together, without overlapping, to give the figure shown below.

The area in square centimetres of this figure is

A 8 cm²

B 10 cm²

38.4 C 12 cm²

D 14 cm²

E 16 cm²

B28.

The total area of the two triangles is

28.9 A 6×8 cm²

B $\frac{6 \times 8}{2}$ cm²

C $\frac{10 \times 6}{2}$ cm²

D $\frac{16 \times 12}{2}$ cm²

E $\frac{20 \times 12}{2}$ cm²

B31. The length of a box was measured and found to be 9 centimetres TO THE NEAREST CENTIMETRE. Which of these could have been the length of the box, measured more accurately?

A 10 cm

B 9.9 cm

C 9.62 cm

D 9.6 cm

58.5 E 8.6 cm

Measurement items - Forms A and B

C2. Which of the following is most likely to be nearest to the weight of a normal man?

A 8.5 kg

65.9 B 85 kg

C 185 kg

D 1150 kg

E 1650 kg

C15. What is the capacity of a cubic container 10 cm by 10 cm by 10 cm?

21.9 A 1 litre

B 10 litres

C 100 litres

D 1000 litres

E 1000 centimetres

C17. Michael has a large number of wooden blocks which are cubical in shape with each edge 1 centimetre long. What is the maximum number of these blocks that can be used to fill a rectangular box with interior dimensions 10 centimetres long, 10 centimetres wide and 7 centimetres high?

A 27

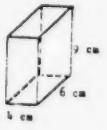
B 70

C 140

D 280

50.4 E 700

C20. What is the SURFACE AREA of this solid rectangular box?



A 50 square centimetres

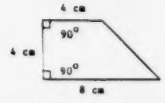
B 100 square centimetres

C 114 square centimetres

D 216 square centimetres

25.8 E 228 square centimetres

C31.



There is a brass plate of the shape and dimensions shown in the figure above. What is its area in square centimetres?

A 16

34.6 B 24

C 32

D 64

E 96

D2. What is the volume of a rectangular box with interior dimensions 10 cm long, 10 cm wide, and 7 cm high?

A 21 cm³

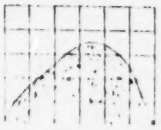
B 70 cm³

C 140 cm³

D 280 cm³

69.2 E 700 cm³

D7.



Each of the small squares in the figure is 1 square unit. Which is the best estimate of the area of the shaded region?

A 10 square units

B 12 square units

57.0 C 14 square units

D 16 square units

E 18 square units

D9. The shaded circle is what part of the area of the large circle?

A $\frac{1}{6}$

B $\frac{1}{5}$

70.3 C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

D23. How many pieces of pipe each 20 metres long would be required to construct a pipeline 1 kilometre in length?

A 5

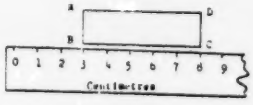
49.2 B 50

C 500

D 5000

E 50,000

D30.



According to the scale shown, the length of side BC of a rectangle ABCD (to the NEAREST CENTIMETRE) is

78.9 A 5 centimetres

B 6 centimetres

C 7 centimetres

D 8 centimetres

E 9 centimetres

Measurement items - Forms C and D

A11. A team scores an average of 3 points per game over 5 games. How many points altogether were scored in the 5 games.

A $\frac{3}{5}$

B $\frac{5}{3}$

C 3

D 5

82.6 E 15

A12.

Test Score	Tally	Frequency
4	/	1
5	///	3
6	//// /	6
7	//	2
8	////	4
9	///	3
10	/	1

The table shows scores for a class on a 10-point test. How many in the class made a score GREATER than 7?

A 2

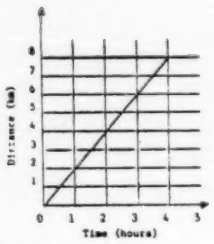
45.8 B 8

C 10

D 12

E 20

A19.



The graph shows the distance travelled by a tractor during a period of 4 hours. How fast is the tractor moving?

A 1 kilometre per hour

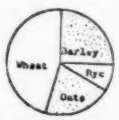
66.3 B 2 kilometres per hour

C 4 kilometres per hour

D 8 kilometres per hour

E There is not enough information.

A22.



The circle graph shows the proportions of various grain crops produced by a country. Which of the following statements is TRUE?

89.0 A More oats than rye is produced.

B The largest crop is barley.

C Equal quantities of wheat and barley are produced.

D The smallest crop is oats.

E Wheat and oats together make up less than half the total grain crop.

B3. The petals on 100 flowers of different kinds were carefully counted, and the results are shown in this table.

No. of Petals	Frequency
10-12	5
13-15	22
16-18	48
19-21	18
22-24	7

How many of the flowers had FEWER than 19 petals?

A 48

B 52

C 73

55.1 D 75

E 93

B4. There are 7,000,000 girls under the age of 21 in a country with a total population of 36,000,000. If a circle graph were drawn showing the distribution of the population, the angle in the sector representing girls under the age of 21 would have measure

A 7°

B 20°

C 21°

18.2 D 70°

E 72°

B9. Joe had three test scores of 78, 76 and 74, while Mary had scores of 72, 82 and 74. How did Joe's average compare with Mary's?

A Joe's was 1 point higher

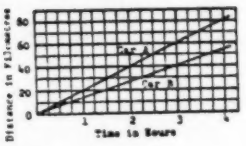
B Joe's was 1 point lower

76.7 C Both averages were the same

D Joe's was 2 points higher

E Joe's was 2 points lower

B19.



How much longer does it take for Car B to go 50 kilometres than it does for Car A to go 50 kilometres?

58.1 A 1 hour 15 minutes

B 1 hour 30 minutes

C 2 hours

D 2 hours 30 minutes

E 2 hours 35 minutes

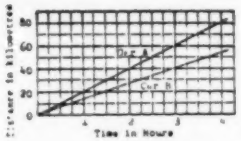
C6. There are five black buttons and one red button in a jar. If you pull out one button at random, what is the probability that you will get the red button.

A 0
42.5 B 1/6
C 1/5
D 5/6
E 1

C7. You wish to know whether SLOSH is the most popular soft-drink in your school. Which of the following ways will give you results you can be most sure of?

A note the number of empty SLOSH bottles in the trash cans.
B ask the manager of the snack bar how many cases of SLOSH he has ordered in the last month.
C ask your friends whether they think that SLOSH is the most popular soft-drink.
D discuss with the driver of the soft-drink delivery truck what he thinks of SLOSH.
82.0 E keep a record of soft-drink sales in the school by brand name over a period of 1 week

C10.



Three hours after starting, car A is how many kilometres ahead of car B?

A 2
B 10
C 15
50.7 D 20
E 25

C27.

In the graph, rainfall in centimetres is plotted for 13 weeks. The average weekly rainfall during the period is approximately

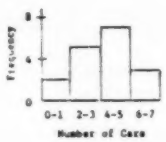
A 1 centimetre
52.1 B 2 centimetres
C 3 centimetres
D 4 centimetres
E 5 centimetres

C30. The arithmetic mean (average) of: 1.50, 2.40, 3.75 is equal to

A 2.40
44.8 B 2.55
C 3.75
D 7.65
E None of these

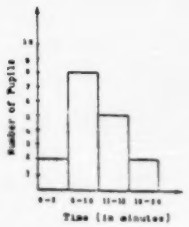
D8. Here is a table of data and a graph of the same data. What is z?

Number of Cars	Frequency
0 or 1	2
2 or 3	z
4 or 5	7
6 or 7	3



A 2
B 3
C 4
63.9 D 5
E 6

D11.



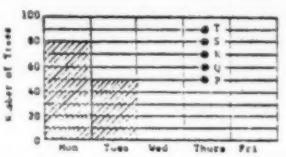
The graph shows the time of travel by pupils from home to school. How many pupils must travel for MORE than 10 minutes?

A 2
B 5
58.8 C 7
D 8
E 15

D34. Here is a table that shows the number of trees planted along a Highway in a week

Days of the Week	Mon	Tues	Wed	Thurs	Fri
Number of Trees Planted	80	50	60	90	75

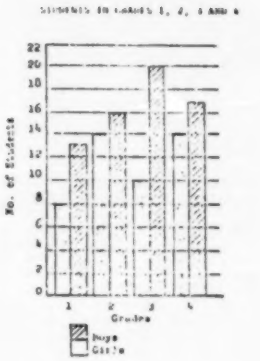
On the diagram below, the graph for the first two days' plantings has been drawn.



If the graph were completed, which point would indicate the top of the bar on Thursday?

A P
B Q
C R
D S
71.8 E T

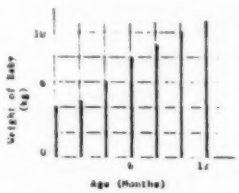
D21



Which of these is a TRUE statement about the information shown on the graph?

A Grade 2 is the smallest class.
B Grades 2 and 4 have the same number of students.
87.6 C Grade 3 has twice as many boys as girls.
D Grade 4 has more girls than boys.
E Grade 1 has as many boys as there are girls in Grade 4.

D35.



The weight gain from 6 to 10 months was

A 1 kg
52.6 B 2 kg
C 4 kg
D 6 kg
E 8 kg

A16.

x	3	6	P
y	7	Q	35

The table above shows the values of x and y, where x is proportional to y. What are the values of P and Q?

A P = 14 and Q = 31

B P = 10 and Q = 14

C P = 10 and Q = 31

D P = 14 and Q = 15

20.6 E P = 15 and Q = 14

A23. The price of an article was $100.00. The price was first raised by 10% and was then reduced by 10% of the new price. What is the price of the article now?

A $90

17.8 B $99

C $100

D $101

E $110

A25. A car takes 15 minutes to travel 10 kilometres. What is the speed of the car?

A 30 kilometres per hour

48.7 B 40 kilometres per hour

C 60 kilometres per hour

D 90 kilometres per hour

E 150 kilometres per hour

B1. 72% is equal to

A 7200

B 720

C 72

D 7.2

62.3 E 0.72

B23. If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30 gram portion of that food?

67.5 A 90

B 100

C 900

D 1000

E 9000

C14. Candidate A received 70 percent of the votes cast in an election. If 4200 votes were cast in the election, how many votes did Candidate A receive?

A 2800

B 2900

45.9 C 2940

D 3000

E 4130

C18. If the ratio of 2 to 5 equals the ratio of n to 100, then n is equal to

A 10

B 20

68.7 C 40

D 150

E 250

D14. In a school election with three candidates, Joe received 120 votes. Mary received 50 votes, and George received 30 votes. What percent of the total number of votes did Joe receive?

A $\frac{6}{10}$%

B 40%

58.6 C 60%

D 80%

E 120%

D32. $150 is divided in the ratio of 2 to 3. The smaller of the two amounts is

A $30

B $50

24.2 C $60

D $90

E $120

APPENDIX G

PRESCRIBED CURRICULUM - MATHEMATICS 8

POPULATION A - MATHEMATICS GRADE 9

COURSE OUTLINE

I. SETS AND SET OPERATIONS

The student is able to:

1. use the following symbols
   a) {a, b, c, ...}
   b) $\in \not\in \subset \cap \cup$
2. state or demonstrate whether two sets are in one-to-one correspondence;
3. specify the elements of a set when given a word description of the set;
4. specify, draw, or state
   a) the union of two or more sets
   b) the intersection of two or more sets.

II. NUMBER AND NUMBER OPERATIONS

A. Notation

The student is able to:

1. distinguish between the concepts of number and numeral;
2. state the value represented by each digit in a multi-digit decimal numeral;
3. write decimal numerals in expanded form using
   a) powers of ten
   b) powers of ten in exponent form;
4. use the common mathematical symbols;
5. write decimal numerals in scientific notation.

B. Number

The student is able to:

1. identify the elements belonging to the following sets of numbers;
   a) natural or counting numbers
   b) whole numbers
   c) rational numbers
   d) integers
   e) irrational numbers
   f) real numbers

2. describe the relationship between the sets of numbers listed above in terms of union and intersection;
3. list the factors of a number;
4. list the multiples of a number;
5. identify the elements belonging to specified sets of numbers: (e.g. prime numbers, even numbers, odd numbers, etc.).

C. Operations

The student is able to:

1. perform the operations of addition, subtraction, multiplication and division with:
   a) whole numbers
   b) rational numbers in
      i. fraction form
      ii. decimal form
      iii. mixed numeral form
   c) integers;
2. round a decimal numeral to a specified place value;
3. perform sight multiplication and division with powers of ten;
4. write a whole number as a product of its prime factors;
5. calculate the G.C.F. of two or more whole numbers;
6. calculate the L.C.M. of two or more whole numbers;
7. write sets of equivalent fractions;
8. write an improper fraction for a mixed numeral and conversely;
9. write a decimal numeral for a fraction numeral;
10. write a fraction numeral for a decimal numeral;
11. write a percent numeral for a fraction numeral or a decimal numeral;
12. use a table to find the approximate square root of a number;
13. perform, in conventional order, a calculation involving a series of operations;
14. solve simple open sentences.

D. Principles

The student is able to:

1. recognize and use the following principles:
   a) commutative for addition and multiplication

b) associative for addition and multiplication
c) distributive
d) addition of zero
e) multiplication by one
f) reciprocals (e.g. $x \cdot \frac{1}{x} = 1 \rightarrow 3 \cdot \frac{1}{3} = 1$
g) opposites (e.g. $x - -x = 0 \rightarrow 5 - -5 = 0$)
h) subtraction (e.g. $x - y = x + -y \rightarrow 7 - 13 = 7 + -13$).

III. GEOMETRY

The student is able to:

1. describe or give the meaning for the concept of congruency for:
   a) segments
   b) angles
   c) triangles;
2. use compass and straight-edge to:
   a) copy an angle
   b) bisect an angle
   c) bisect a segment
   d) construct a perpendicular to a line at a point on a line
   e) construct a perpendicular to a line from a point not on the line
   f) construct a parallel to a line through a given point
   g) copy a triangle or construct a triangle to specifications;
3. identify from a diagram, or draw a diagram illustrating:
   a) adjacent angles
   b) complementary angles
   c) supplementary angles
   d) vertical angles
   e) obtuse angles
   f) right angles
   g) acute angles
   h) isosceles triangles
   i) equilateral triangles
   j) scalene triangles
   k) right triangles
   l) parallel lines
   m) altitudes of triangles
   n) quadrilaterals
      i. parallelograms
      ii. rectangles
      iii. squares
   o) parts of a circle;
4. calculate the measure of an angle of a triangle given the measures of the other two angles;
5. use the Pythagorean Theorem to calculate any side of

right triangles.

IV. MEASUREMENT

The student is able to:

1. describe the steps involved in the measurement process; namely
   a) choosing a unit
   b) dividing the object to be measured into the specific units
   c) counting the number of units used;
2. use a ruler to measure length in metric units;
3. use a protractor to measure angles;
4. state and use the metric units of
   a) length
   b) angular measure (degrees)
   c) area
   d) volume;
5. perform the basic arithmetic operations using units of measure.

V. PROBLEM SOLVING

The student is able to:

1. analyze a verbal problem and
   a) distinguish between what is given and what is to be found
   b) recognize whether sufficient information is given to solve the problem
   c) determine what operations are required to solve the problem;
2. construct a "model" such as a diagram as an aid to solving a problem;
3. translate verbal problems into open sentences;
4. construct a flow chart to fit a verbal description of an operation or procedure;
5. test the appropriateness of an answer to a problem.

VI. GRAPHS AND FUNCTIONS

The student is able to:

1. calculate the output number when given input numbers and a function rule;

2. graph on a number line:
   a) whole numbers
   b) integers
   c) rational numbers in
      i. fraction form
      ii. decimal form
3. graph the solution set of an open sentence on the number line

VII. APPLICATIONS OF MATHEMATICS

The student is able to:

1. draw and interpret scale diagrams;
2. solve problems involving percent, (e.g. interest, discount, sales tax, etc.);
3. calculate perimeters and areas of:
   a) rectangles
   b) triangles
   c) circles
   d) parallelograms
   e) other polygons;
4. calculate volumes of:
   a) rectangular prisms
   b) cylinders.

VIII. LOGICAL THINKING

The student is able to:

1. solve open sentences involving the connectives "and" and "or".

TEXTBOOKS:

a) Eicholtz, et al.: School Mathematics II (Addison-Wesley)
b) Sobel, Maletsky: Mathematics II (Ginn)
c) Sobel, Maletsky: Essential of Mathematics II (Ginn)

POPULATION B - ALGEBRA 12

COURSE OUTLINE

TOPIC Note: Numbers refer to the Scope and Sequence Chart	ALGEBRA 12 CORE FORMAT	ENRICHED FORMAT	NOTES Note: References to Dolciani in Alg. 12 are intended to support and supplement the treatment of topics provided by the texts UAA and PCM.
4. Extending the Number System (complex numbers)	+, -, ×, ÷ of complex numbers $a + bi$ complex numbers as ordered pairs, graphing UAA CH. 5, pp. 176-194 Dolc CH. 10, pp. 398-409	As in core	1. This topic is in Algebra 11 enriched. 2. The real number system and operations with radicals in UAA CH. 5 may be reviewed and supplemented with questions from Dolc, pp. 258-267.
5. System of Linear Sentences (3 equations in 3 unknowns)	Solution of 3 equations in 3 unknowns UAA CH. 6, pp. 210-217 Dolc CH. 3, pp. 95-107	As in core	1. This topic is in Algebra 11 enriched. 2. Linear equations and their graphs may be reviewed and supplemented with questions from Dolc, pp. 90-94.
7. Quadratic Relations	Discriminant of quadratic Nature of roots UAA CH. 8, pp. 289-292 Dolc CH. 7, pp. 273-279 Dolc CH. 10, pp. 407-409	As in core	1. This topic is in Algebra 11 enriched.
8. Trigonometry	The six trigonometric functions and their graphs (see notes 1, 3) Solution of right triangles (see note 2) Law of sines Law of cosines Solving oblique triangles (see note 4) Vectors (see note 4) Trigonometric identities Sum and difference formulas Even and odd functions Double and half-angle formulas Conditional trigonometric equations Graphs of $y = A \sin (nx + \alpha)$ etc. (see notes 5, 6) UAA CH. 12, pp. 426-457 UAA CH. 13, pp. 458-478 Dolc CH. 10, pp. 363-398 Dolc CH. 11, pp. 413-431 and pp. 436-448 Dolc CH. 12, pp. 455-469 and pp. 477-483	The circular functions (see note 3) Solution of right triangles (see note 2) Law of sines Law of cosines Solving oblique triangles (see note 4) Vectors (see note 4) Trigonometric identities Sum and difference formulas Even and odd functions Double and half-angle formulas Conditional trigonometric equations Graphs of $y = A \sin (nx + \alpha)$ etc. Inverse trigonometric functions Principal values Areas of sectors and segments Trigonometric form of complex numbers De Moivres theorem nth roots of complex numbers PCM CH. 5, pp. 125-164 PCM CH. 6, pp. 176-193 Dolc CH. 10, 11, 12	1. The words "circular function" and the relationship of trigonometric functions and their graphs to the unit circle should be stressed in reviewing the material already taken in Alg. 11 Core. Radian measure should receive more emphasis. 2. Solution of right triangles is in Alg. 11 Core and is reviewed here. The appropriate geometry of similar triangles, Pythagoras' theorem should be reviewed. 3. The graphs of the three basic trigonometric functions are covered in Alg. 11 (core and enriched). The graphs of the three reciprocal functions are added here. The relationship between the circular and trigonometric functions should be made clear. 4. This topic is in Alg. 11 enriched. 5. Good place to review and use translations and dilations. See topic 11, Alg. 12 core. 6. Students should be familiar with the terms amplitude, period and phase shift.

TOPIC Note: Numbers refer to the Scope and Sequence Chart.	ALGEBRA 12 CORE FORMAT	ALGEBRA 12 ENRICHED FORMAT	NOTES Note: References to Dolciani in Alg. 12 are intended to support and supplement the treatment of topics provided by the texts UAA and PCM.
9. Conics and Quadratic Systems The Second Degree Relations	Distance formula Midpoint formula Equation of circle – standard form – general form Equation of parabola – definition – equation $x^2 = 4py$ etc. Equation of ellipse – definition – equation $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ etc. Equation of hyperbola – definition – equation $\frac{x^2}{a^2}-\frac{y^2}{b^2} = 1$ etc. Standard forms of equations of conics (see notes 1, 2) Completion of square, etc. Inverse variation (see note 4) Quadratic systems – graphing – solving Problems UAA CH. 9, pp. 307-349 Dolc CH. 8, pp. 293-330	Same listing as in core (see notes 3, 5) UAA CH. 9, pp. 307-349 PCM CH. 9, pp. 273-285	1. The idea of translation is covered in Alg. 11 in UAA CH. 7. This should be reviewed. 2. Graphs of quadratics are treated in UAA CH. 8. This should be reviewed. 3. The treatment and language in PCM is more formal than that in UAA. Latus rectum, eccentricity are introduced. 4. Inverse variations is part of topic 6, Alg. 11 (core and enriched formats). It should be reviewed in connection with the hyperbola $xy = K$. 5. In Alg. 12 enriched emphasis should be given to translation of axes and students should be shown, using rotation of axes, that every second degree equation represents a conic. Rotation problems need not be done.
10. Exponentials, Logarithms, Transcendental Functions	Rational and real exponents Exponential functions – graphs Logarithmic functions – graphs – properties of logarithms Common logarithms (see notes 2, 3) – calculations using logarithms – interpolation Exponential and logarithmic equations Problems UAA CH. 10, pp. 350-385 Dolc CH. 9, pp. 333-359	Rational and Real Exponents Exponential and transcendental functions (see note 1) – graphs The number e. Binomial theorem and $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$ (see note 4) Growth and decay Logarithmic functions – graphs – properties of logarithms Natural and common logarithms – calculations Equations involving logarithms and exponentials PCM CH. 11, pp. 333-357 Dolc CH. 9, pp. 333-359	1. PCM defines "transcendental" functions. 2. PCM emphasizes natural logarithms, UAA emphasizes common logarithms. 3. UAA emphasizes calculations using logarithms. The prevalence of calculators makes this emphasis unnecessary. 4. This topic should be postponed until after the Binomial theorem in UAA CH. 11, sec. 8 (topic 13).

TOPIC Note: Numbers refer to the Scope and Sequence Chart.	ALGEBRA 12 CORE FORMAT	ENRICHED FORMAT	NOTES Note: References to Dolciani in Alg. 12 are intended to support and supplement the treatment of topics provided by the texts UAA and PCM.
11. Graphing Techniques	Some graphing techniques (see notes 1, 2) – translation – dilation – reflection UAA CH. 7, pp. 251-263	Graphing techniques (see note 3) – excluded regions – symmetry, odd and even – functions – asymptotes Continuity PCM CH. 4, pp. 97-112 and pp. 118-121	1. This material is review. Teachers are referred to note 4 of topic 6 in Alg. 11 outline. 2. It is expected that teachers will have reviewed and used these techniques in dealing with the graphs of the conics, the trigonometric functions and exponential and logarithmic functions. 3. These techniques could be taken in conjunction with graphing polynomial and rational functions in topic 12.
12. Polynomial and Rational Functions	Polynomial defined Synthetic division Remainder and factor theorems Integral roots Rational roots Fundamental theorem of Algebra Graphs of polynomials Complex roots occur in conjugate pairs Descartes Rule of Sign UAA CH. 14, pp. 479-504 Dolc CH. 14, pp. 519-533	Polynomial function Graphing polynomials Division algorithm $P(x) = D(x) + Q(x) + R(x)$ Remainder theorem Factor theorem Fundamental theorem of Algebra Complex roots occur in conjugate pairs Synthetic division Upper and lower bound of roots Integral roots Rational roots Rational functions PCM CH. 10, pp. 297-332 Dolc CH. 14, pp. 519-538	1. Descartes Rule of Sign is not in UAA.
13. Sequences and Series	Arithmetic sequences and series – nth term – means – sum of n terms Summation notation Applications of arithmetic sequences and series Geometric sequences and series – nth term – means – sum of n terms Infinite geometric series – idea of limit – sum as $n \to \infty$ Applications Binomial theorem (see note 2) – Pascal's triangle – factorial notation – indicated term Fibonacci sequences UAA CH. 11, pp. 388-425 Dolc CH. 13, pp. 487-514	As in the core with some selected questions from PCM CH. 13, pp. 391-405 (see note 1)	1. UAA treatment should be used for both core and enriched topics. 2. e as $\lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n$ should be discussed in the enriched format.

TOPIC Note: Numbers refer to the Scope and Sequence Chart.	ALGEBRA 12		NOTES Note: References to Dolciani in Alg. 12 are intended to support and supplement the treatment of topics provided by the texts UAA and PCM.
	CORE FORMAT	ENRICHED FORMAT	
14. Vectors in the Plane		Vector defined — equivalence class operations with vectors $\vec{u} + \vec{v}$ $\vec{u} - \vec{v}$ $c\ \vec{v}$ Axioms of vector space (see note 1) Geometric proofs using vectors (see note 2) Vectors as ordered pairs Magnitude of a vector $\|\vec{v}\|$ Vector projection, components Bases, linear combination, linearly dependent Parametric equation of line (see note 3) Scalar product PCM CH. 7, pp. 197-232	1. Students should not be required to memorize the axioms. 2. This provides a good opportunity to review geometry and to introduce the idea of proof. 3. The concept of parametrization is important in further studies.
15. Polar Coordinates		Coordinates of points (see note 1) Polar distance formula (see note 2) Equations of lines and circles (see note 3) Symmetry Classical curves (see note 4) Points of intersection (see note 5) Relations between polar and rectangular coordinates (see note 6) PCM CH. 12, pp. 363-390	1. Non-uniqueness of coordinates should be stressed. 2. This need not be stressed. 3. General equations need not be stressed. 4. Students should plot points on the curves. Calculators may be used to evaluate r. After familiarity has been gained, sketching of curves should be emphasized. 5. Some problems here are very difficult. They should not be emphasized. 6. This topic is important.
OPTIONAL TOPICS **The optional topics are not outlined in detail. Teachers should choose material of interest to their students and at a suitable level of difficulty.**			
16. Vectors in 3-space. Solid Analytic geometry		(See Notes 1 - 3) PCM CH. 8, pp. 241-272 PCM CH. 9, pp. 290-292	1. This material extends vectors on the plane to 3-space. 2. Students may find visualization difficult. 3. The material in CH. 9 deals with quadric surfaces – extension of the conics into 3-space.

TOPIC Note: Numbers refer to the Scope and Sequence Chart.	ALGEBRA 12		NOTES Note: References to Dolciani in the Alg. 12 are intended to support and supplement the treatment of topics provided by the texts UAA and PCM.
	CORE FORMAT	ENRICHED FORMAT	
17. Linear Algebra		Matrices and determinants (See note 1, 2) UAA CH. 16, pp. 534-570 Dolc CH. 15, pp. 543-572	1. This material is relatively easy. 2. Applications to geometry should be mentioned.
18. Introduction to Calculus		(See notes 1, 2) PCM CH. 14, pp. 427-513	1. The approach in PCM is abstract and students may find it difficult. 2. Except with very able students, teachers should consider using a more intuitive approach.
19. Probability and Statistics		(See note 1) UAA CH. 15, pp. 505-533	1. UAA provides a good introduction to Probability and Statistics. Teachers should, wherever possible, use an experimental approach with examples chosen from newspapers, games, etc.

TEXTBOOKS:

a) Del Grande, et al.: Mathematics For a Modern World 1112 (Gage)
b) Travers, et al.: Using Advanced Algebra (Doubleday)
c) Crosswhite et al.: Pre Calculus Mathematics (Merrill)
d) Dolciani: Modern Algebra and Trigonometry, Book 2 (Nelson)

Queen's Printer for British Columbia ©
Victoria, 1982